

THE  
Young Gentleman's  
COURSE OF  
MATHEMATICKS.

Containing the more Useful and Easy  
ELEMENTS of

|               |   |             |
|---------------|---|-------------|
| Aritbmctick,  | } | Opticks,    |
| Geometry,     |   | Astronomy,  |
| Trigonometry, |   | Chronology, |
| Mechanicks,   |   | Dialling.   |
|               |   |             |

In Three VOLUMES.

By EDWARD WELLS, D. D. Rector  
of Cotesbach in Leicestershire.

L O N D O N.

Printed for James Knapton, at the Crown in St.  
Paul's Church-Yard. 1714.



THE  
Young Gentlemen's

COURSE OF  
MATHEMATICS

Containing the most useful and easy  
METHODS



In Three VOLUMES

By EDWARD HULL, Esq. F.R.S.  
of Cambridge in England

LONDON

Printed for J. & W. TAYLOR, in the Strand  
and for J. & W. TAYLOR, in the Strand

THE  
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MATHEMATICKS.

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*N. B.* This may serve for Directions to Bookbinders, how to place and bind up the several Parts of this Course of *Mathematicks*,

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THE



*Gal 9 Ad*

THE  
Young Gentleman's  
ARITHMETICK,  
AND  
GEOMETRY;

Containing such  
ELEMENTS of the said Arts  
or Sciences, as are most useful  
and easy to be known.

By EDWARD WELLS, D.D.  
Rector of Cotesbach in Leice-  
stershire.

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THE

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ARITHMETICK

AND

GEOMETRY

Containing

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By EDWARD WELLS D.D.  
Rector of Christ Church in Oxford  
1713

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THE  
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ARITHMETICK;

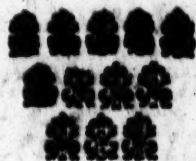
CONTAINING

The more Easy and Useful  
ELEMENTS of *Arithmetick*,  
both *Common* and *Algebraical*.

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By EDWARD WELLS, D. D.  
Rector of Cotesbach in Leiceſter-  
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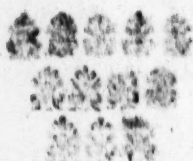
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THE  
 Young Gentleman's  
 ARITHMETICK  
 CONTAINING

The more Easy and Useful  
 ELEMENTS of Arithmetick  
 both Common and Algebraical.

BY EDWARD D. D.  
 Regor of Coleridge in the  
 University of Cambridge.  
**MUSEVM  
 BRITANNICVM**



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*The General*

**PREFACE.**

**A***nd it is an Happiness of Men above Brutes, that they are endued with Reason; so it is an Happiness of Gentlemen above the meaner Part of Mankind, that, being freed from the Common and Bodily Employments of Life, they have Leisure to exercise their Reason in more Noble Studies: Among which may justly be reckon'd the Study of Mathematicks.*

## The General Preface.

For as it is one Branch of  
~~the Transcendent Excellency of~~  
**GOD**, ~~that He is the infinitely~~  
Wise Creator of all Things;  
so it is one Branch of the Ex-  
cellency of Man, that He is  
able to contemplate and ap-  
prehend the infinite Wisdom  
of his Creator, manifested in the  
Works of the Creation: Where-  
unto Nothing conduces more than  
the Knowledge of Mathema-  
ticks. For we are assured (\*)  
by the wisest of Men, that  
**GOD** has order'd all Things  
in Measure, Number, and  
Weight, that is, according to  
the Rules of Mathematicks:

---

(\*) Wisdom of Solomon, Chap. II. Ver. 20.

And

## The General Preface.

And therefore, He that would rightly apprehend the Excellent Order or Contrivance of the Creation, must first understand the Sciences relating to Measure, Number, and Weight, that is, the several Parts of Mathematicks.

And, since G O D sends no One into the World, to be idle, or only to take his Pastime therein; but the more He has free'd Gentlemen from bodily Labour, the more He expects they should exercise the Faculties of their Minds, in order to his greater Glory, by raising their Minds to more clear and sublime Apprehensions of his Divine Perfections; and since

## The General Preface.

the Mathematical Sciences are so necessary to the Attainment of such Apprehensions of the Divine Excellency: On these Considerations it may not unjustly be look'd on, as one Part of the Gentleman's Calling, to apply a due or considerable Portion of his Time, to the Study of Mathematicks.

For to affirm this, is Nothing else but to affirm in other Words, that it is the Duty of a Gentleman, to allot a considerable Portion of his Time, (which, by the Bounty of GOD in giving him a good Estate, is exempted from mean and bodily Employments,) for Qualifying himself to glorify his great Crea-

tor



## The General Preface.

tor and Bountiful Benefactor, with the most exalted Nations he can; namely, by applying himself (as to Books of Religion, which tend in a more special Manner to set forth the Divine Wisdom, as well as Goodness, in the Redemption of Mankind; so also) to Books of Mathematicks, which tend in a more special Manner to set forth the Divine Wisdom and Goodness, in Making and Preserving the System or Frame of the Material World.

These Considerations had a principal Share, in inducing Me to draw up this Course of Mathematicks for Young Gentlemen: Forasmuch as it hence appears

## The General Preface.

appears to be a *Work*, which in its own Nature tends very much to the Promoting of *GOD's* Glory, and consequently to be a *Work* very proper for a Divine.

And as what has been said, do's, with Respect to my self, carry in it a sufficient Justification for my Drawing up this Course of *Mathematicks*; so with Respect to Others, it carries in it such a Motive to the Study of *Mathematicks*, as will be of the greatest Weight with Young Gentlemen, who are of a Sober and Religious Disposition.

As for those, who are not to be influenc'd by Motives of a Spiritual Nature, or drawn from their Duty to God, consider'd  
either

## The General Preface.

either as their common Creator, or special Benefactor, there are not wanting many sensible Motives, or such as may be drawn from their temporal Interest, to encourage Them to the Study of the Mathematical Sciences. Some of these Motives are expressly taken Notice of, in the particular Prefaces to the several Treatises, which make up this Work: And others may be easily gather'd from the Nature of many Examples contain'd in the Treatises themselves. I shall therefore, in this General Preface, content my self with taking Notice of this General Advantage, arising to a Gentleman from his Knowledge of the Mathematicks: Namely, that He

is

## The General Preface.

is thereby enabled to entertain Himself at any Time, (either without Door if the Weather be fair, or within Door if it be foul,) after such a Manner, as shall either beneficially instruct, or innocently divert his Mind; and consequently He needs never be at a Loss how to pass away his Time as he ought, That most unhappy Condition, which has prov'd the fatal Ruin of so many Gentlemen, as to Body as well as Soul, as to their Temporal as well as Eternal State.

It remains only to observe, that another general Encouragement to Young Gentlemen for to Study the Mathematicks, and that no inconsiderable One, is  
the



## The General Preface.

*the Method observ'd in drawing up these Treatises; wherein are selected only such Elements as are most useful and easy to be known. By which Means they may attain to a competent Knowledge of the Mathematicks, with much less Labour, and in much less Time, in going through this Course of Mathematicks, than in going through Others, wherein the more and less useful, the easy and difficult Elements are contain'd promiscuously together. For what more might be said here, I refer the Reader to the Preface to the Astronomy hereunto belonging, as being the first of these Treatises which was drawn up and publish'd.*

**The**

The General  
INTRODUCTION.

I.  
Mathema-  
ticks, why  
so call'd.

THE Word *Mathematicks* do's  
in the Greek Language, from  
which it is deriv'd, denote or im-  
ply so much as *Sciences* chiefly to  
be learned or studied; namely, by  
Reason of their great Use, not only  
in the speculative and more abstruse  
Parts of Learning, but also in Pra-  
ctice, or the common Concerns of  
Life.

2.  
The Object  
of Mathe-  
maticks,  
what.

The Object of *Mathematicks*, is  
*Quantity*; which may, either be  
computed or numbered, and then  
it is call'd *Multitude* or *Number*; or  
measur'd, and then it is call'd *Mag-  
nitude*.

3.  
Pure Ma-  
thema-  
ticks,  
what.

The Mathematical Science, which  
treats of *Number*, is call'd *Arith-  
metick*; and that which treats of  
*Magnitude*, is call'd *Geometry*.  
(One

## *The General Introduction.*

(One Part whereof is peculiarly stil'd *Trigonometry*, as treating of the Measuring of Triangles.) Both *Arithmetick* and *Geometry* are distinguish'd from the other Mathematical Sciences, by the Name of *Pure Mathematicks*; forasmuch as they treat of Number and Magnitude, as consider'd purely in themselves, or abstractedly from all physical or natural Operations depending thereon.

The other Mathematical Sciences are in general call'd by the common Name of *Mixt Mathematicks*; forasmuch as they treat of Bodies under a *mixt* Consideration, viz. partly Mathematical, and partly Physical or Natural; that is, they explain Physical Operations by Mathematical Principles, viz. by the Principles of *Arithmetick* and *Geometry*.

4.  
Mixt Ma-  
thema-  
ticks,  
what.

Thus

## The General Introduction.

5.  
Mechanicks,  
Opticks,  
&c. why  
accounted  
mixt Ma-  
themat-  
icks.

Thus *Mechanicks* by Mathematical Principles explain the Motion of Bodies, and the Force of Machin's or Engin's: *Opticks*, by the same explain the Manner of Vision or Sight: *Astronomy*, the several Phænomena or Appearances of the Celestial Bodies: *Chronology*, distinguishes and adjusts the Variety of Times: And *Dialling* calculates the Distance of the Hour-lines, whereby Time is measur'd. In like Manner *Geography* is esteem'd a Part of mixt Mathematicks, as adjusting the true Situation of Places, &c. by Mathematical Principles: *Navigation*, as thereby calculating the Situation of the Place where the Ship is at any Time, and the like: And lastly, *Musick*, as treating of Sounds with Respect to their various Quantity or Measure, that is, Mathematically. Of these mixt Mathematicks, *Mechanicks*, *Opticks*, *Astronomy*, *Chronology*, and *Dialling*,  
are



## The General Introduction.

are treated of in this Course of  
Mathematicks. *Geography* I have  
treated of in a Treatise publish'd  
by it self many Years ago. The  
other two, *Navigation* and *Musick*,  
though reckon'd as Part of mixt  
Mathematicks, yet are not of so  
general an Use, and consequently  
are omitted in this and other  
Courses of Mathematicks.

(b) THE

---

T H E  
P R E F A C E.

**W**HEN I drew up the Young Gentleman's Astronomy, I had little or no Thoughts to proceed to any other Parts of the Mathematicks, than those Two, which have a more immediate Dependence on Astronomy, viz. Chronology and Dialling. But the same Motives, which induced me to draw up those Treatises, prevailing afterwards upon me to go through a Course of Mathematics after the same Manner; as it was necessary for me to draw up a Treatise of Arithmetick, so I have adapted it to the peculiar Design of the said Course of Mathematicks, taking Notice only, or chiefly of such Particulars, as are most requisite for Young Gentlemen to know.

Succ

## The Preface.

Such as are of so commendable a Disposition, as not to be content with barely knowing so much of Arithmetick, as is requisite to the common Concerns of Life; but to desire to understand further the Grounds of Arithmetick, and its more curious and abstruse Parts; may to this End peruse my Latin Arithmetick, publish'd formerly for the Use of Young Students in the Universities.

It may very likely be a Matter of some Difficulty to procure it, the Edition being many Years since in a Manner quite gone of, and so this Difficulty not to be remov'd but by a New Edition.

He must be very little concern'd with the common Affairs of Life, that is not sensible of the great Usefulness of Arithmetick. However, because it is not impossible, but some Young Gentlemen may be so Brisk and Airy, as to think, that the knowing how to cast Accompt is requisite only for such Underlings as

(b 2)

Shop-

## The Preface.

Shop-keepers or Trades-men, but unnecessary and below Persons of plentiful Estates; I therefore judge it very proper to convince them of their great Mistake in this Point, by a late Instance of a Gentleman, who, (as I have been accidentally, but credibly inform'd,) having a considerable Estate seiz'd upon for Debt, and being desir'd by his Friends (who were much surpriz'd at this his Misfortune, as not knowing him to be guilty of any Extravagancy) to tell them, by what Means so great a Misfortune befel him; assur'd them, that it was principally owing to his not knowing how to cast up or keep his Accompts. For being unwilling to discover this his Ignorance, he had been forced to take such Accompts as others brought him, without examining them; and thereby he had given them Opportunity, to cheat him out of his Estate. So that this Instance is a sufficient Proof and Warning, that it is not enough for a Gentleman, to take  
Care



## The Preface.

Care not to run out his Estate himself by any extravagant Way of Living, unless he knows also how to take Care, and actually do's take Care, by Casting up and Keeping, or at least Examining his Accompts, that his Estate do's not run from him, through the Knavery of others; and consequently, that no Gentleman ought to think Arithmetick below Him, that do's not think an Estate below Him.

THE

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T H E

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(1)

THE  
Young Gentleman's  
ARITHMETICK,

BOTH  
*Common and Algebraical.*

CH A P. I.

*Of Notation, or the Art of Expressing Numbers by their right Characters, and of Reading them aright, when so expressed.*

**A**RITHMETICK, like the Names of the other liberal Arts or Sciences, is of Greek Original, and imports, in that Language, as much as *the Art of right Numbring.*

Right Numbring consists, either in Denoting (or Expressing) Numbers by their right Characters, which is called *Notation*, or in

B

on,

general  
Parts, No-  
tation and  
Computa-  
tion.

on, and comprehends under it the Art of Reading Numbers aright, (forasmuch as he that can denote Numbers aright, can read them aright, when so denoted;) and in Computing aright by Adding Numbers together, or Subtracting them one from the other, and the like; which may therefore be called *Computation*.

3.  
Notation  
two-fold.

Notation is either *Literal* or *Figural*.

4.  
Literal No-  
tation,  
what.

*Literal* Notation is so called, because it expresses Numbers by Letters.

5.  
Literal  
Notation  
used by the  
eastern  
Nations.

It was formerly, if not still, made use of by most of the eastern Nations, as the *Hebrews* or *Jews*, *Chaldeans*, *Syrians*, *Arabians*, *Persians*, &c. who all expressed their Numbers by the Letters of their respective Alphabets.

6.  
Of the No-  
tation of  
the Greeks.

The *Greeks* did likewise express Numbers by the several Letters of their Alphabet. They had also another Way of expressing Numbers by the initial capital Letters of some of their Numeral Words, as  $\Pi$  Πέντε *Five*, Δ Δέκα *Ten*, Η Ημετέρι *an Hundred*, Χ Χίλιαι *a Thousand*, Μ Μύρια *ten Thousand*.

7.  
Of the No-  
tation of  
the Latins,  
or Romans.

From this last Method of expressing Numbers used by the *Greeks*, the *Latins* in all Probability deduced their Notation by the initial capital Letters of some of their numeral Words, and other Characters.

## Arithmetick.

Characters made from the said Capitals, viz.  
*V Five, X Ten, L Fifty, C an Hundred,*  
*D five Hundred, M a Thousand.*

Among the *Latins* as well as *Greeks*, 8.  
 the Character *I*, denotes *One*; it being (\*) *I*, the natural Character of *One*, and therefore usual even for the most illiterate Persons, to denote one single Thing, by one single Stroke or Line. and is used both by the *Greeks* and *Latins*.

*M*, As being the initial (or first Letter) 9.  
 of *Mille*, is the *Latin* Character for a *Thousand*. And whereas the old Way of writing this Letter was thus *CD*, (for which our Printers now a-days use *CD*, hence Half the said Letter, *D*, came to be used by the *Latins* for Half a *Thousand*, or *five Hundred*. And because this same half Letter resembles a *D*, or the Letter *I* with a *C* turned the wrong Way; hence Scribes and Printers have used the said *D* or *CD* (instead of the said half *D*) to denote *five Hundred*.

---

(\*) One single Stroke or Line being the natural and most simple Character of one single Thing, there was no occasion for the *Greeks* or *Latins* to make use of the initial capital Letter of their numeral Word for *One*. And it is evident, the *Latins* did not. As for the *Greeks* it is commonly thought, that they took *I* for to denote *One*, as being the Initial of *Is*, which signifies *One* in their Language; but it seems much more natural to suppose, that the Word *Is* was used by them to signify *One*, because *I* is the natural Mark for *One*; and that *Is* was made from not *I* taken from *Is*.

## The Young Gentleman's

4

**10.** In like manner C, being the Initial of *Centum*, denotes in *Latin* an *Hundred*. And whereas the said Letter was anciently writ thus, E, hence Half of it, viz. L, was used to denote among the *Latins* Half an *Hundred* or *Fifty*: Which half Letter resembling a Capital L, Printers scrupled not to use the Letter L, instead thereof to denote *Fifty*.

**11.** According to the same Method it seems very probable, that D, as being the Initial of *Decem*, did at first denote among the *Latins* *Ten*; and Half of the said Letter D, viz. D, did thence denote half *Ten* or *Five*. Which said half Letter resembling somewhat a V, therefore Printers (and Scribes) have not scrupled to put the Letter V (instead of the Letter D halved) for *Five*. And forasmuch as the whole Letter D came (as has been observed, *Señ. 9.*) to be used for *five Hundred*; therefore, there has been another Character found out for *Ten*, viz. two half D's joined with their Bottoms together, or two V's joined at the Points of narrow Ends together, and so resembling an X, hence used to denote twice *Five* or *Ten*.

**12.** Thus much for the Original of the *Roman* or *Latin* Notation to a *Thousand*. As for the *Millenary* Characters, where by they expressed Numbers above a *Thousand*.



# Aritbmetick.

Thousand, they were deduced from the letter **CD**, as is shewn in the following Table.

## A T A B L E of the Numeral Characters used by the Romans or Latins.

13.  
A Table of the numeral Characters used by the Latins.

One.  
Five.  
Ten.  
Fifty.  
an Hundred.  
or **ID** or **Io** five Hundred.  
or **CID** or **cIo** a Thousand.  
**IOO** five Thousand.  
**CCIOO** ten Thousand.  
**CCO** fifty Thousand.  
**CCCIOO** an hundred Thousand.  
**DOO** five hundred Thousand.  
the **CCCCIOO** a Million.

By these Characters variously joined together or repeated, the *Latins* were wont to express their Numbers. Where it is to be observed, that

14.  
Observations concerning the Latin Notation.

1. Characters of greater Value, are regularly and generally placed before Characters of less Value. Thus **VI** denotes Six, **XI** Eleven, **LX** Sixty, **CK** an Hundred and Ten, **MDCCXII** (the present Year of Christ) one Thousand seven Hundred and Twelve.

15.  
Observation on the 1st.

2. **I** and **X** are sometimes placed before Characters of greater Value, namely, **B 3** **I** before

16.  
Observation on the 2d.

## The Young Gentleman's

I before V or X, and X before L or C in which Case, (†) the Value of I and X is to be subtracted from the Value of the following Character. As IV *Four*, IX *Nine*, XL *Forty*, XC *Ninety*.

17.  
*Observation  
on the 3d.*

3. V and L are never repeated, and none of the other Characters above four Times. Thus IIII (or IV) *Four*, but V *Five*; XXX *Thirty*, but XL *Forty*; LXXX *Eighty*, but XC *Ninety*; CCCC *four Hundred*, but D *five Hundred*. And thus much for the *Roman* or *Latin* Notation of Numbers.

18.  
*Of the Algebraical  
Notation by  
Letters.*

In reference to literal Notation, it remains now only to observe, that there is another numeral Use of Letters, namely *Algebraical*; according to which any Letter is put to denote any Number. Concerning which, see more *Chap. 9*.

19.  
*Of the nine  
Numeral  
Figures.*

Proceed we here to figural Notation or the Way of expressing Numbers by *Figures*; whereby are understood the Characters commonly made use of to denote Numbers among us, and are the *Nine*, viz. 1 *One*, 2 *Two*, 3 *Three*, 4 *Four*, 5 *Five*, 6 *Six*, 7 *Seven*, 8 *Eight*, 9 *Nine*.

---

(†) Some use IIX instead of VIII to denote *Eight*, and XXX instead of LXXX to denote *Eighty*, &c.

In order to perceive, wherein lies the great Difference between literal and figural Notation, it is to be known, that in all Numbers (considered even in themselves, and without any Respect to the Things number'd) are distinguishable two Parts. Whereof one may be called the *denominative* Part, as being that which tells us, *what* the numeral Quantity is which is number'd, whether Units, or Tens, or Hundreds, &c. The other may be called the *numerative* Part, as telling *how many* Units, or Tens, or Hundreds, (&c.) are denoted. Thus in the Number *ten Thousand*, *Thousand* is the *Denominative*, *Ten* is the *numerative* Part; in *twenty* (i. e. *two Tens*) *Tens* is the *Denominative*, *Two* the *Numerative*; and so in the least Number of all, viz. *one* (i. e. *one Unit*) *Unit* is the *denominative* Part, *One* the *Numerative*.

This being premised, the great Difference between literal and figural Notation consists in this, that numeral Letters are *restrained* always to denote one and the same, both *denominative* and *numerative* Part of the Number, which they express; whereas numeral Figures are *restrained* always to denote only the same *numerative* Part, and vary as to the *denominative* Part, according to the various Places wherein they stand. For Instance: The

Every Number consists of two Parts, a Denominative and Numerative.

21. The great Difference between literal and figural Notation wherein it consists.

Letter V always denotes five Units, where as the Figure 5 denotes five Units, or Tens, or Hundreds, (&c.) according to the Place wherein it stands; as 555 denotes *five Hundred fifty-five*.

22. *Figural Notation more expedite than Literal.*

Hence arises the more expedite Manner of expressing Numbers by Figures, than by Letters. For Instance: The present Year of our Lord requires seven Letters to express it, viz. MDCCXII; whereas it may be expressed only by these four Figures 1712. And in expressing some other Numbers, there is a much greater Difference; as for Example, MDCCCLXXVII 1777, MDCCCLXXXVIII 1888.

23. *A Table shewing the Method of Figural Notation.*

Now Figures thus (always retaining only their numerative Value, but) changing their denominative Value, according to the Place or Rank wherein they stand hence the whole Art of figural Notation is comprehended in the following Table.

| Trillions.                             |        | Billions.                              |        | Millions.                              |        | Units.                                 |        |
|----------------------------------------|--------|----------------------------------------|--------|----------------------------------------|--------|----------------------------------------|--------|
| Thas.                                  | Units. | Thas.                                  | Units. | Thas.                                  | Units. | Thas.                                  | Units. |
| 800,000,000,000                        |        | 800,000,000                            |        | 800,000                                |        | 800                                    |        |
| HTU, HTU, HTU, HTU, HTU, HTU, HTU, HTU |        | HTU, HTU, HTU, HTU, HTU, HTU, HTU, HTU |        | HTU, HTU, HTU, HTU, HTU, HTU, HTU, HTU |        | HTU, HTU, HTU, HTU, HTU, HTU, HTU, HTU |        |

24. *The Places of Figures how reckoned.*

In the fore-going Table are these Particulars observable.

1. That numeral Places are reckoned from the Right-hand, and so contrary to the Order wherein the Figures are read.



So that as there are in all twenty-four Places in the Number express'd in the Table, so the Figure 2, that is outmost to the Right-hand, is to be esteemed the first as to Notation; and the Figure 2, that is outmost to the Left-hand, is to be esteemed the last Figure in the whole Number.

2. To every three Figures are orderly repeated the Denominations of *U, T, H*, i. e. *Units, Tens, Hundreds*. Thus every 222 denote *two Hundred, twenty (i. e. two Tens), two (i. e. two Units)*. Hence it comes to pass, that whosoever can read three Figures together, may with a little more Instruction be (||) quickly able to read any Number, of how many Figures soever it consists. For the Number is to be read, as distinguished into so many three Figures, beginning from the Right-hand as in the Table.

3. To every three Figures from the Right-hand, the Names, *Units and Tens, Sands*, are alternately applied. And to every six Figures from the Right-hand, a new general Name is given. As to the first six Figures the general Name of *U*.

(||) It can be experimentally said, that several Young Gentlemen have (by the Method here explained) been taught in a very few Minutes Time, to read Sums of 20, 40, 60, &c. Figures.

with

is given, to the second six Figures the general Name of *Millions*, to the third six Figures the general Name of *Billions*, to the fourth six Figures the general Name of *Trillions*, &c. Hence every six Figures, reckoning from the Right-hand, is called a *Period*, and every Period is distinguished into two half Periods. Now by a *Billion* is meant, in short, what is otherwise called a *Million of Millions*, and by a *Trillion*, is meant a *Million of Millions of Millions*. And the like is to be understood of *Quadrillions*, *Quintillions*, *Sextillions*, &c.

27.  
[An Observation concerning the Pronouncing or Reading of Numbers.]

4. Though the Name *Units* is regularly to be understood, as applied to every half Period alternately, and also to the whole first Period, yet 'tis not usual to pronounce the same in reading Numbers. Thus the first three Figures 222 are usually read thus, *two Hundred twenty-two*, without adding *Units of Units*. And in like Manner, the third three Figures, 222, are usually read thus, *two Hundred twenty-two* (not *Units of Millions*, but simply) *Millions*.

28.  
[The foregoing Observations or Rules concerning figural Notation, illustrated.]

By the fore-going Observations duly apprehended, it will be easy to read the Number consisting of twenty-four Figures, and contained in the Table of figural Notation, (Sect. 23,) namely, by first dividing the said Number into its

*Periods*

Periods by full Stops, and then into its illustrated by an Example.  
 half Periods by Comma's, as above in the  
 respective Table, and then pronouncing  
 each three Figures distinctly with their  
 proper Denominations, thus:

Two Hundred Twenty-two Thousand,  
 two Hundred Twenty-two, Trillions.

Two Hundred Twenty-two Thousand,  
 two Hundred Twenty-two, Billions.

Two Hundred Twenty-two Thousand,  
 two Hundred Twenty-two, Millions.

Two Hundred Twenty-two Thousand,  
 two Hundred Twenty-two, (Units.)

In like manner, the following Number 29.  
 of twenty Figures, viz.

97.546.832.172,945.631,874, is to be And again by another Example.  
 read thus:

Ninety-seven, Trillions.

Five Hundred Forty-six Thousand,  
 eight Hundred Thirty-two, Billions.

One Hundred Seventy-two Thousand,  
 nine Hundred Forty-five, Millions.

Six Hundred Thirty-one Thousand,  
 eight Hundred Seventy-four (Units.)

Where it is to be observed, that, as in  
 this, so in other Numbers, it frequently  
 happens, that the last Period (or half  
 Period) is often not compleat.

From what has been said concerning 30.  
 figural Notation, it evidently appears, Of the Use of Cyphers,  
 that as the numerative Values of the nine  
 Figures are known by their Shape, viz.

1 One,

1 *One*, 2 *Two*, 3 *Three*, (&c.) so their denominative Values depend upon their Places, and are therefore variable. Thus in the Number expressed in the Table, (set down in Sect. 23.) the Figure 2 in the first Place denotes two *Units*, in the second Place two *Tens*, in the third Place two *Hundreds*, in the fourth Place two *Thousands*, &c. Hence it appears, that in order to denote just two *Tens* or *Twenty*, without any *Units*, it is necessary to shew some how or other, that the Figure 2 stands not in the first Place, but in the second Place. And to this End serves (what is called) a (\*) *Cypher*; namely, to fill up such antecedaneous Places, as are vacant of significant Figures, and thereby to shew the respective Places of the significant Figures. Thus 2 (put by it self, and so understood to be in the first Place) denotes two *Units*; but 20 (i. e. 2 with a *Cypher* afore it, whereby is shewn, that the said 2 stands in the second Place) denotes two *Tens* or *Twenty*. In like manner, 200 denotes two *Hundreds*; and 2,000 two *Thon-*

---

(\*) Hence it is most probably derived from an *Arabic* Word, denoting a *Vacancy*. It is by Way of Distinction termed by some the *Insignificant* Figure, as signifying no Number it self; the other Nine being termed *significant* Figures.



*sands*; and 2.000,000 two *Millions*, &c. But one or more Cyphers set after (i. e. on the Left-hand of) a significant Figure, do not promote (or increase) the denominative Value of the said Figure, because they do not promote the said Figure into an higher Place, but it still continues in the same Place with respect to Notation, as it did afore the Cyphers were adjoined thereto. Thus 02, and 002, and 0002, and 0000002, (&c.) does each denote no more than 2, viz. two Units.

As what has been said concerning figural Notation, if rightly apprehended, is abundantly sufficient to teach any one of a competent Capacity, how to read any Number proposed, and already expressed by Figures; so is it in like manner abundantly sufficient to teach, how to express by Figures any Number proposed. For Instance: Suppose it be required to express *twenty two Millions*. From what has been said, it is known, that the Figures 22 denote *Twenty-two*. And, because *twenty-two Millions* is a shorter Way of speaking (as has been observed, *Señ. 27.*) for *twenty-two Units of Millions*; and by the Table of figural Notation, (*Señ. 23.*) it is known, that *Millions* is the general Name of the second Period, and *Units of Millions* the Name of the first half Period.

Period of the said second Period ; hence it is known also, that 22 must stand in the two first Places of the second Period, i. e. in the 7th and 8th Places from the Beginning, and consequently must have six Cyphers set before it, (namely, to fill up all the six Places of the first Period, which in this Number is void of significant Figures,) to make it denote *twenty-two Millions* ; which is therefore denoted thus, 22 000 000.

32.  
Integers or  
whole  
Numbers,  
and Fra-  
ctions,  
what.

And thus much for Notation (Literal as well as Figural) of *Integers* or *whole Numbers*. Where it is to be observed, that Numbers are so called, when the Things number'd are consider'd as so many whole Things ; whereas should the same Things be consider'd as Fragments or Parts of any Thing else, then the Number is call'd a *Fraction*. For Example : Ten Shillings consider'd by themselves, or as so many whole Shillings, is esteem'd a whole Number ; but ten Shillings consider'd as ten Twentieths, or the Half of a Pound, is esteem'd a *Fraction*.

CHAP. II.

Of Computation, and the four Primary Operations of Arithmetick in general.

PROCEED we now to the second and much larger Part of Arithmetick, <sup>2.</sup> *To compute or cast Account, is to find a Number sought, having two or more Numbers given.* called *Computation*. *To compute then, or* <sup>pure or cast Account, what,</sup>

And this is done, either singly by Adding together the Numbers given, or by <sup>3.</sup> *Computations, an addition, or subtraction,* Subtracting one from the other, or by <sup>4.</sup> *into simple or primary Operations,* Multiplying or Dividing one by the other; or else, lastly, by some two or more of these Operations jointly. Hence this second Part of Arithmetick is distinguished into *simple or primary Operations*, and *compound* (viz. of the Simple, two or more) or *secondary Operations*. <sup>and compound or Secondary.</sup>

The simple or primary Operations are <sup>3.</sup> *The primary Operations Four.* Four, viz. *Addition, Subtraction, Multiplication, and Division.*

In like manner, the secondary Operations are reducible to Four in general, <sup>4.</sup> *The secondary Operations, likewise reducible to Four in general.* viz. *Reduction, such as relate to Proportion, Evolution, or Extracting the Roots of Powers, and Equation.*

As

5. The primary and secondary Operations to be spoken of, only in reference to Figural and Algebraical Computation.

As it has been observed, (*Chap. 1. Sect. 22.*) that figural Notation is much shorter and quicker than Literal; so it is obvious to conceive from thence, that figural Computation is much more easy than Literal: Insomuch, that though literal Notation is still used in some Cases, yet literal Computation is quite laid aside, excepting only as to its *Algebraical Use*. For this Reason it will be needful to speak of the primary and secondary Operations of *Arithmetick*, only in reference to *Figural and Algebraical Computation*.

6. Of the Characters or Signs of the four primary Operations.

To begin then with the four primary Operations, with reference (\*) to figural Computation. Each of the said Operations is expressed in short by a certain Character. Thus + is the Sign of *Addition*, - of *Subtraction*, \* of *Multiplication*, and )( or ÷ of *Division*. To which add = the Sign of *Equality*. Hence,

$$\begin{array}{lcl}
 2+4=6 & \left. \begin{array}{l} \text{denotes in words} \\ \text{thus much, viz.} \end{array} \right\} & \begin{array}{l} \text{Two added to Four, is equal to Six} \\ 6-4=2 & \left. \begin{array}{l} \text{6 when 4 is subtr. from it, is equ. to 2} \\ 4 \times 2=8 & \left. \begin{array}{l} \text{Four multiplied by Two, is equal to 8} \\ 2)8(4 \text{ or } \frac{8}{2}(4 & \left. \begin{array}{l} \text{Eight divided by Two, is equal to 4} \end{array} \right\} \end{array} \right\}
 \end{array}$$

(\*) What is here said, does in good Measure relate also to *Algebraical Computation*, as is shewn *Chap. 9.*



Having shewn the Characters or Signs, whereby each of the four primary Operations is denoted in short, I proceed now to lay down such Rules, as relate in general to the four primary Operations.

**7.** General Rules relating to the four primary Operations.

Numbers of the same (whether (+) intrinsecal or extrinsecal) Denomination must be placed directly one under another; as Units under Units, Tens under Tens, &c. Pounds under Pounds, Shillings under Shillings, Pence under Pence, &c. Yards under Yards, Feet under Feet, Inches under Inches, Years under Years, Days under Days, Hours under Hours, &c.

**8.** Rule the 1st.

The particular Sum of any Denomination is always to be resolved into as many following or higher Denominations, as may be. Thus 24 Units are to be resolved into two Tens and four Units, and each Figure to be set in its proper Place according to Rule the 1st, viz. 4 under Units or in the first Place, 2 under Tens or in the second Place. In like manner, 59 Farthings are to be resolved

**9.** Rule the 2d.

(+) By *intrinsecal* or *internal* Denominations, are meant such as belong to Numbers considered in themselves, as *Units, Tens, Hundreds, &c.* By *extrinsecal* or *external* Denominations, are meant such as belong to the Things numbered, as *Pounds, Shillings, Pence, &c.*

C

into

# The Young Gentleman's

into 1 Shilling, 2 Pence, and 3 Farthings, and the Figure 1 to be placed under Shillings, 2 under Pence, and 3 under Farthings.

**TO.** *Addition, Subtraction, and Multiplication* begin at the Right-hand Figures, and go on to the Left: *Division* on the contrary.

**II.** *The best Proof of Addition is Subtraction, of Multiplication by Division; and on the contrary, Subtraction best proved by Addition, and Division by Multiplication.* For what *Addition* and *Multiplication* join together, *Subtraction* and *Division* disjoin again. For Example: Because  $3 + 4 = 7$ , therefore  $7 - 4 = 3$  or  $7 - 3 = 4$ . And in like manner, because  $2 \times 3 = 6$ , therefore  $2 \overline{)6}$  (3, or  $3 \overline{)6}$  (2). And thus much for what relates to the four primary Operations in general, proceed we now to speak of each in Particular.

(1) For there are other Sorts of Proof, among which the more usual is by casting away 9, of which I shall give an Instance in each Chapter belonging particular to the several Operations.

# CHAP. III.

## Of Addition of Integers.

**A**ddition gathers the several Numbers given into one Sum, all along following the general Rules laid down in the fore-going Chapter.

### EXAMPLE I,

of Numbers of one (external) Denomination.

$$\begin{array}{r} 9857634208 \\ 1134587692 \\ \hline \end{array}$$

The Sum 10992221893

### EXAMPLE II,

of Numbers of one (external) Denomination.

$$\begin{array}{r} 24670578 \\ 42009871 \\ 2405060 \\ 6432 \\ 10567891 \\ 7021600 \\ 540 \\ \hline \end{array}$$

The Sum 86681972

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## EXAMPLE III,

Of Numbers of several (external) Denominations, viz. of Money.

|                | <i>l.</i>    | <i>s.</i> | <i>d.</i> | <i>q.</i> |
|----------------|--------------|-----------|-----------|-----------|
|                | 1200         | 14        | 06        | 2         |
|                | 9231         | 19        | 11        | 0         |
|                | 0000         | 10        | 10        | 2         |
|                | 562          | 12        | 11        | 1         |
|                | 400          | 00        | 00        | 0         |
|                | 129          | 18        | 06        | 2         |
|                | 800          | 09        | 07        | 0         |
| <b>The Sum</b> | <u>12326</u> | <u>07</u> | <u>04</u> | <u>3</u>  |

## EXAMPLE IV,

Of Numbers of several Denominations, of Measure.

|                | <i>Yards.</i> | <i>Feet.</i> | <i>Inch.</i> |
|----------------|---------------|--------------|--------------|
|                | 94            | 2            | 3            |
|                | 165           | 1            | 9            |
|                | 700           | 0            | 8            |
| <b>The Sum</b> | <u>960</u>    | <u>1</u>     | <u>8</u>     |



**EXAMPLE V.**

*Numbers of several Denominations, viz. of Time.*

|               |              |               |             |
|---------------|--------------|---------------|-------------|
| <i>Years.</i> | <i>Days.</i> | <i>Hours.</i> | <i>Min.</i> |
| 1000          | 1941         | 06            | 200         |
| 12            | 210          | 18            | 50          |
| 174           | 000          | 11            | 000         |

The Sum 197 (\*) 187 14 12

The best or surest Way of proving, whether any of the fore-going Examples be rightly done, is (according to the 4th general Rule) by *Subtraction*, as shall be shewn in Chap. 4. And this Proof is universal, holding Good in respect to Numbers of several (external) Denominations, as well as of One. As for Numbers of one (external) Denomination, the *Addition* of them may be proved with good Certainty enough another Way; namely, having cast away 9, as often as you can, both out of the Numbers given to be added, and also out of

2.  
Of the  
Proof of  
Addition,  
by casting  
away 9  
often as  
may be  
done.

(\*) The Sum of the Days do amount to 352; of which 3 make up a common Year, and the remaining Days will be 187. If the Sum of the Days arise to four Years, then 366 Days must be allowed for one (viz. the Leap) Year.

the Sum arising from thence, if both the Remainders be the same, it may be look'd upon as sufficiently prov'd, (at least in Matters of no important Concern,) that the Addition is rightly performed. For Instance, 9 being cast away as often as you can, out of the two Numbers given in Example the First there will remain 1, or One; and likewise 9 being cast away, as often as you can, out of the Sum, there will remain One. Whence it may be look'd upon (+) sufficiently prov'd, that the Addition is duly performed in the said Example.

3.  
Another  
Example  
of the same  
Sort of  
Proof.

In like manner, 9 being cast away, as often as may be, both out of the Numbers given to be added in Example the Second, and also out of the Sum, the Remainders of both will be the same viz. 2. Whence it may with (+) Certain

(++) The Reason why this Sort of Proof is not infallible, (as is the other by Subtraction,) is, because the Remainders in this Case will both be the same, as long as the Figures be the same, though they be never so much alter'd as to the Places of them; and consequently, though the Numbers be different from the Numbers given, or the Number to be found. As will appear by comparing Example here subjoin'd with Example the First.

5789622410  
296785431

6086417841

they enough be inferr'd, that the Addition  
be duly performed also in Example the  
(second. And so much for the Addition  
of Integers, or whole Numbers, accor-  
ding to Algebraic Computation,

Subtraction takes the less Number  
out of the greater, and to find the  
Difference between the two Numbers  
even, or (as it is otherwise called) the  
Remainder or Residue of the  
greater Number above the less.

The general Rules are all along  
observed, Rules which it is to be  
observed also, that the greater Number  
to be placed regularly above the less.  
Moreover it is to be observed, that  
although it is (\*) common to subtract

CHAPTER

of Numbers, out of which it is to  
be subtracted; yet it often happens, that  
Figure (one or more) of the upper Num-  
ber may be less than that Figure of the  
lower Number, which is to be subtracted  
from it. In which Case the said  
Figure is to be increased by adding thereto  
One taken from the next higher (the  
following) Denomination, and then the

1.  
Subtraction  
out, what

2.  
A portion  
of the  
Rule  
relating to  
Subtraction  
on

Another  
particular  
Rule  
relating to  
Subtraction  
in which

(\*) In Algebraic Subtraction it is not necessary  
that the less Number be taken from the greater.

## CHAPTER IV.

## Of Subtraction of Integers.

1.  
Subtraction,  
on, what.

**S**ubtraction takes the less Number given out of the Greater, and so finds the Difference between the two Numbers given, or (as it is otherwise called) the Residue or Remainder or Excess of the greater Number above the less.

2.  
A particular Rule  
relating to  
Subtraction.

The general Rules are all along to be observed. Besides which it is to be observed also, that the greater Number is to be plac'd regularly above the less.

3.  
Another  
particular  
Rule relating to  
Subtraction.

Moreover it is to be observed, that although in (\*) common *Arithmetick* the Number to be subtracted, taken all together, must be always less than the upper Number, out of which it is to be subtracted; yet it often happens, that Figure (one or more) of the upper Number may be less than that Figure of the lower Number, which is to be subtracted from it. In which Case, the said less Figure is to be increased by adding thereto One taken from the next higher (i. e. following) Denomination, and turn'd in

(\*) In *Algebraick Subtraction* this is not necessary, may be seen Chap. 9.



to an equivalent Number of the same Denomination with the said left Figure, And in the Process of the Work, the next upper Figure is to be esteem'd less by One than it is express'd, by Reason of the One thus afore taken from it. For Instance : Let it be propos'd to take four Shillings and ten Pence out of twelve Shillings and two Pence. The Numbers must be writ, as on the Side hereof. Now because 10d cannot be subtracted out of 2d; therefore the 2d is to be increased by adding thereto (one taken from the next higher Denomination, viz.) 1s, one Shilling, and turn'd into an equivalent Number of Pence, viz.) 12d, whereby the 2d will become 14d, out of which may be subtracted the 10d, and there will remain (as on the Side) 4d. And so the Remainder of the whole will be 7s. 4d, forasmuch as the 1s in the Process of the Work, is to be esteem'd no more than 11s, by Reason of the 1s taken from it, and added to the 2d to make it 14d. And (as on the Side) 11s. - 4s. = 7s.

$$\begin{array}{r} 12\text{ s. } 02 \\ - 4\text{ s. } 10 \\ \hline 07\text{ s. } 04 \end{array}$$

If

4.  
A third  
particular  
Rule rela-  
ting to Sub-  
straction.

If it so happens, that the next higher Denomination has no significant Figure (as in the Example on the Side,) then the 2d must however be increas'd as a-fore. And in like manner, there being no Shillings in the upper Number, there must be taken One from the next higher Denomination, i. e. one Pound, and out of it must be deducted the 4s of the lower Number. Only it must be re-member'd, that, whereas the one Pound, being turn'd into Shillings, does make 20 Shillings, they are here to be esteem'd but 19 Shillings, (as is exprest on the Side,) by Reason of the one Shilling taken a-fore to increase the 2d to 14d. And in like manner, the 3l. is to be esteem'd but 4l.; and the whole Substraction to be so perform'd, as if the Numbers were written as on the Side in the last Instance.

| l.    | s. | d. |
|-------|----|----|
| 3     | 00 | 00 |
| 1     | 04 | 10 |
| <hr/> |    |    |
| 3     | 15 | 00 |
| <hr/> |    |    |
| 4     | 19 | 10 |
| 1     | 04 | 10 |
| <hr/> |    |    |
| 3     | 15 | 00 |

5.  
Another  
Instance in  
reference  
to the fore-  
going par-  
ticular  
Rules.

In like manner in Numbers of one (external) Denomination, One taken from the next higher (internal) Denomination, is equal to Ten of the next lower (internal) Denomination, forasmuch as the

Said

# Arithmetick

the said internal Denominations ex- 705  
ceed one the other by a decuple 27  
Proportion. And if there be a 678  
Cypher in the next higher (inter-  
nal) Denomination, then in the  
Process of the Work, the One taken  
from the Figure following the Cypher,  
must be esteem'd (not 10, as otherwise it  
should have been, but) 9, namely, by  
Reason of the One taken  
before. Hence the fore-go- H. T. U.  
ing Example of one exter- 6 . 9 . 15  
nal Denomination, if ex- 2 . 7  
press'd according to the man- 6 . 7 . 8  
ner of Working, will stand  
as on the Side, viz. the up-  
per Number 705, must be conceiv'd  
as denoted thus, 6 Hundreds,  
9 Tens, and 15 Units, which is equivalent to  
705 (+).

(+) There is another Method commonly made use of in  
this Case. Which is by equally increasing both the Num-  
bers given; namely, by adding as many to the left upper  
Figure, as are equal to one of the next higher Denomina-  
tion; and also by adding one (or, as it is commonly call'd,  
paying the one borrow'd) to that  
Figure of the next higher Denomi-  
nation, which is to be subtracted,  
as on the Side. But this Way being  
not so natural, the Reason of it is  
not so easily to be apprehended as  
of the other, which is therefore  
here preferr'd.

|    |    |    |    |
|----|----|----|----|
| s. | d. | s. | d. |
| 8  | 6  | 8  | 16 |
| 1  | 7  | or | 2  |
| 1  | 7  |    | 7  |
| 1  | 9  |    | 9  |

What

6. What is said being rightly apprehended, it will be easy to perform all the following Examples of *Subtraction*. Which will also serve to shew, how *Addition* is proved by *Subtraction*, and *Subtraction* by *Addition*; the Example I. of *Subtraction*, answering to Example I. of *Addition*, and so of the Rest.

Examples  
of Subtra-  
ction.

EXAMPLE I,

Of Numbers of one (external) Denomination.

|                              |             |                                           |
|------------------------------|-------------|-------------------------------------------|
|                              | 10992221893 | The Sum in Addition                       |
|                              | 9837634201  | } The Numbers given to be added together. |
|                              | <hr/>       |                                           |
| The Difference or Remainder, | 1134587692  |                                           |

EXAMPLE II,

Of Numbers of one (external) Denomination.

|               |          |                                                                      |
|---------------|----------|----------------------------------------------------------------------|
|               | 86681972 | The Sum in Addition                                                  |
|               | 24670578 | } One Number given to be added.                                      |
|               | <hr/>    |                                                                      |
| The Remainder | 62011394 | The Equivalent (or Sum) of the rest of the Numbers given to be added |



# Arithmetick.

## EXAMPLE III.

Of Numbers of several (external) Denominations, viz. of Money.

| l.    | s.   | d.   | q.  |                             |
|-------|------|------|-----|-----------------------------|
| 12326 | . 07 | . 04 | . 3 | The Sum in Add.             |
| 1200  | . 14 | . 06 | . 2 | One Num. given to be added. |
| <hr/> |      |      |     |                             |

The Remaind. 11125 0120 10 : 1 The Equivalents of the rest.

## EXAMPLE IV.

Of Numbers of several (external) Denominations, viz. of Measure.

| T.    | F.  | In. |                               |
|-------|-----|-----|-------------------------------|
| 960   | . 1 | . 8 | The Sum in Addition           |
| 94    | . 2 | . 9 | One Number given to be added. |
| <hr/> |     |     |                               |

The Remaind. 865 . 2 . 5 The Equivalents of the rest.

## EXAMPLE V.

Of Numbers of several Denominations, viz. of Time.

| Y.    | D.    | H.   | M.   |                             |
|-------|-------|------|------|-----------------------------|
| 197   | . 187 | . 14 | . 12 | The Sum in Add.             |
| 10    | . 341 | . 06 | . 20 | One Num. given to be added. |
| <hr/> |       |      |      |                             |

The Residue 186 . 211 . 07 . 52 The Equivalents to the rest.

7.  
How to  
subtract  
several  
Numbers  
from One.

If more than one Number is to be subtracted out of the same greater Number, then it is expedient to add together all the Numbers to be subtracted into one Sum, and so to subtract the said Sum For Instance.

Receiv'd 12325.07.04.3

9231.19.11.0

Sum of Money paid  
away.

Paid away  
at several  
Times.

000.10.10.2

552.13.11.1

400.00.00.0

129.18.06.2

800.09.07.0

= 1125.12.12.1

Remains 1200.14.06.2

And this last Example, compar'd with Example III, both of Addition and Subtraction, is a further Evidence, how the one is proved by the other.

8.  
The Proof  
of Substra-  
ction by  
casting  
away 9.

It remains only to observe, that the Subtraction of Numbers of one (external) Denomination, may be presum'd to be rightly perform'd, if the Remainder of the greater Number be the same as the Remainder of the other Numbers (viz. the Number or Numbers to be subtracted, and the Residue) 9 being cast away.

way as often as may be. Thus as to  
Example I, of Subtraction.

$$\begin{array}{r} 10992221893 \quad 1 \dots 1 \\ 9857634201 \quad 2 \dots 1 \\ \hline 1134587693 \quad 3 \dots 1 \end{array}$$

And thus much for the Subtraction of  
Integers.

CHAP.

## Of the Multiplication of Integers

1.  
Multipli-  
cation,  
what.

**M**ultiplication is in Reality no other than a different Sort of Addition for to multiply one Number by another, is nothing else, but, to add one Number given to it self, so often as the other Number given directs. Thus  $4 \times 3 = 4 + 4 + 4 = 12$ .

2.  
Multipli-  
cand, Multi-  
plicator,  
and Pro-  
duct, what.

The Number added to it self or multiplied, is called the *Multiplcand*, the other, the *Multiplicator* or *Multiplier* (both these together, the *Factors*;) and the Number arising from the Multiplication, the *Product*.

3.  
The first  
particular  
Rule relat-  
ing to Mul-  
tiplicati-  
on.

Of the two Numbers given, either may be the *Multiplcand* or *Multiplicator* (for the *Product* will be the same, as  $4 \times 3 = 12 = 3 \times 4$ ;) but it is better and more usual, to take the less Number for the *Multiplicator*, and to place it under the *Multiplcand*.

4.  
The se-  
cond par-  
ticular  
Rule.

If the *Multiplicator* consists of more than one Figure, then the whole *Multiplcand* is to be added to it self, first so often as the Right-hand Figure of the *Multiplicator* shews, then as often as the next Figure of the *Multiplicator* shews, &c.



and so on. Thus  $421 \times 23$ , is equal to  $1 \times 3$ , and also  $421 \times 20$ .

The Product arising from each Figure of the Multiplier, multiplied into the whole Multiplicand, is to be plac'd by it self in such manner, that the first or right-hand Figure thereof may stand under that Figure of the Multiplier, from which the said Product arises. For instance :

5.  
The third particular Rule.

Multiplicand  
Multiplier

421  
23

Particular Product of  $421 \times 3$

1263

Particular Product of  $421 \times 20$

842

The Total Product

9683

The Reason of so placing the right-hand Figure of each particular Product, in Conformity to the first general Rule, inasmuch as the right-hand Figure of each Product, is always of the same Denomination with that Figure of the Multiplier, from which it arises. Thus in the fore-going Example, the Figure 3 in the Product 1263, is of the Denomination of Tens, as well as the Figure 3 in the Multiplier. For  $1 \times 20$  (i. e. the 1 of 23) = 20, or 2 put in the Place of Tens, or second Place.

D

If

6.  
The fourth  
particular  
Rule.

If either of the Numbers given have one or more Cyphers at the Right-hand the *Multiplication* may be perform'd without taking Notice of the Cyphers, till the Product of the other Figures, be found : To which are to be prefixt, on the Right-hand, the Cyphers before not taken Notice of. For Instance :

|                                                       |                                                              |                                                       |                                                           |
|-------------------------------------------------------|--------------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------------|
| $\begin{array}{r} 12 \\ 10 \\ \hline 120 \end{array}$ | $\begin{array}{r} 358 \\ 6000 \\ \hline 2148000 \end{array}$ | $\begin{array}{r} 10 \\ 10 \\ \hline 100 \end{array}$ | $\begin{array}{r} 2400 \\ 30 \\ \hline 72000 \end{array}$ |
|-------------------------------------------------------|--------------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------------|

7.  
The fifth  
particular  
Rule.

If the Multiplier has any Cypher intermixt with its other Figures, they need not be taken Notice of at all. Thus,

$$\begin{array}{r} 8012 \\ 3000 \\ \hline 48078 \\ 40000 \\ \hline 40118078 \end{array}$$

8.  
Of the Mul-  
tiplicati-  
on Table.

In *Multiplication* it is of great Use readily to know what is the Product of any two of the nine Figures, otherwise called Digits. To which End there is drawn up a Table, from its Use, called the *Multiplication Table*. This is so con-  
triv'd

hav'd, that of the two Digits given to multiply'd, one being found in the upper Row, the other in the left-hand Row of the Table, their Product is contain'd in that Cell, which is made by the meeting of the Lines drawn from the said Digits given. Thus  $6 \times 4 = 24$ ,  $9 = 81$ , &c. as in the Table follow.

**The Multiplication Table.**

| 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---|----|----|----|----|----|----|----|----|
| 2 | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 3 | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

The several Columns of the fore-going Table, being wrote on so many Rods or Pieces of Past-board or Wood, or like, separate one from the other, like what they call *Naper's Bones*, which are very useful for quick and true Work in *Multiplication*. For the Rods, whose top Figures express any Multipli-

D 2

cand

of Naper's Bones or Rods.

and given, being plac'd duly together in their Order, (which is call'd in our Word, the *Tabulating* of the Multiplicand,) shew all the Multiples or Products of the Multiplicand, arising (from any of the 9 Digits, and consequently) from the Multiplier given. So, that nothing more need be done, than to transfer from the said Rods the several Products arising from the given Multiplicand, at each Digit or Figure of the given Multiplier.

10.  
How to  
read and  
trans-  
cribe a-  
right the  
Products  
exhibited  
by Naper's  
Rods.

And, in order to transcribe or read right the said Products, it is to be observed, 1<sup>st</sup>, That the lower Triangle of the first or right-hand Rod, belongs to each of the 9 Digits, do's always belong to the first Place of the Product to be transcrib'd; the upper Triangle of the Last or Left-hand Rod, do's always belong to the last Place of the said Product: The intermediate Rhombs belong to the intermediate Places in their respective Order. 2<sup>dly</sup>, The Figures, that are in the same Rhomb, are to be added together. And if their Sum can't be express'd but by two Figures, the first Figure must be writ in that Place, which the Rhomb belongs; the second Figure must be carried to the following Rhomb or Triangle, and added to the Figure





11.  
Of Multi-  
plication  
by Loga-  
rithms.

There is also another compendious  
Way of Multiplying large Numbers  
namely, by the Help of *Logarithms*, con-  
cerning which I shall speak in the last  
Chapter of this Treatise, wherein the  
Use of *Logarithms* is explain'd.

12.  
Of multi-  
plying  
Numbers  
of several  
external  
Denomina-  
tions, by a  
Multipli-  
cator of  
One Fi-  
gure.

Hitherto I have spoken of the Multi-  
plication of Numbers, only of *one* exte-  
nal Denomination. As for the Multipli-  
cation of Numbers of *several* external  
Denominations, if the Multiplier be a  
single Figure, it has no Difficulty in it  
but is easily perform'd according to the  
general Rules, as in the following Ex-  
ample: Where, supposing seven Persons  
are to be paid each 245 *l.* 6 *s.* 10 *d.* each  
it is required to tell what Sum in the  
whole will pay the same; the Answer  
will be 1717 *l.* 7 *s.* 10 *d.* For,

| <i>l.</i> | <i>s.</i> | <i>d.</i> |
|-----------|-----------|-----------|
| 245       | 6         | 10        |
|           |           |           |
| 1717      | 7         | 10        |

13.  
By a Mul-  
tiplicator  
of more  
figures  
than One.

If the Multiplier consists of more than  
one Figure, then the Operation is not  
so easy. And the more usual Way  
in this Case is to reduce the Multiplicand

sever

several external Denominations into one external Denomination: Of which therefore we shall speak, *Chap. 10*, which treats of *Reduction*.

But there is also another Way which may be used in this Case, and is much shorter than that by *Reduction*: Namely, by resolving the Multiplier given into such Digits, as are, when multiply'd one into the other, equivalent to the Multiplier, and so multiplying by each of the said Digits singly. Thus, supposing twenty-seven Persons were each to be paid 245 *l.* 6 *s.* 10 *d.* and I would know, what Sum is requisite to pay the same, because  $9 \times 3 = 27$ , therefore I multiply 245 *l.* 6 *s.* 10 *d.* first by 9, and then the product thereof by 3; and the last Product will be the Sum sought, and the same as will arise by multiplying 245 *l.* 6 *s.* 10 *d.* (reduc'd into one Denomination) by 27. As will appear by comparing the following Example, with the Example contain'd in *Sect. 7*, of *Chap. 10*, concerning *Reduction*.

24.  
Another  
Way.

D 4

245

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 245 \cdot 6 \cdot 10. \\ \hline \end{array}$$

$$\begin{array}{r} 2208 \cdot 1 \cdot 6 \\ \hline \end{array}$$

$$\begin{array}{r} 6624 \cdot 4 \cdot 6 \\ \hline \end{array}$$

15.  
Of Multi-  
plicators of  
several ex-  
ternal De-  
nominati-  
ons.

As for Multipliers of several external Denominations, they never occur but when the external Denominations are of various Measures, as Yards, Feet, Inches, &c. And in this Case, the Operation may be perform'd, by reducing both the Multiplicand and Multiplier to one Denomination, as is shewn, §. Chap. 5, of *Reduction*.

16.  
Further,  
concerning  
the same.

There is also another Method in the last Case, which is called *Cross Multiplication*. But this, as well as *Reduction* depending on *Division*, it will be improper to speak of it here, before we have spoken of *Division*. And therefore shall be referr'd likewise to Chap. Where it shall be added by Way of Annotation, to the *Reduction* belonging to this Case.



The best or most certain Way of proving, whether *Multiplication* be perform'd right, is (according to the fourth general Rule) by *Division*, as shall be shewn in the following Chapter of *Division*. It remains only here to shew, how the *Multiplication* of Numbers of one external Denomination, may with certainty enough for common Matters be prov'd by casting away 9. Namely, 9 being cast away, as often as may be, out of the Multiplicand, and likewise distinctly out of the Multiplier, multiply the two Remainders one into the other, casting away also 9 out of what arises, if it be large enough. The Remainder thence arising, will be equal to the Remainder of the Product, after 9 is cast away therefrom, as often as may be; if the *Multiplication* prov'd hereby be rightly perform'd. For Instance, in the first Example of this Chapter, the Remainders will be 8, and in the fifth Example will be 6, as appears by the said Examples here repeated.

Of proving Multiplication by casting away 9.

$$\begin{array}{r} 421 \dots 7 \\ 23 \dots 5 \\ \hline 9683 \dots 8 \end{array} \qquad 7 \times 5 = 35 \dots 8$$

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$$\begin{array}{r} 8013 \dots 5 \\ 3006 \dots 2 \\ \hline 40113078 \dots 6 \end{array}$$

And this may suffice for the *Multiplication of Integers.*

**CHAPTER**

## CHAP. VI.

### Of the Division of Integers.

**D**ivision shews, how often the Divisor is contain'd in the Dividend; whence the Number found by this Operation, is called the Quotient. The several Numbers are plac'd, either

thus, Divisor  $3 \overline{)6}$  (2 Quotient, or thus, Dividend  $6 \overline{)3}$  (2 Quotient. And the O-

peration begins from the Left-hand Figure or Figures of the Dividend, according to the third general Rule.

What is properly Division, is perform'd by the Quotient's shewing, how often the Divisor is contain'd in, or may be taken out of the Dividend; as  $3 \overline{)6}$  (2, and when the Divisor does thus exactly divide the Dividend, it is call'd an aliquot Part of the Dividend. Thus 3 and 2 are aliquot Parts of 6.

But because it frequently happens, that the Divisor is not exactly contain'd in the Dividend, but after the Division there remains somewhat of the Dividend; therefore in order to know what the said Remainder is, the Quotient must be multiplied

1.  
Division,  
what. As  
also the  
Dividend,  
Divisor,  
and Quoti-  
ent.

2.  
An Ali-  
quot Part,  
what. As  
also Multi-  
plication  
and Sub-  
traction,  
why requi-  
site to Di-  
vision.

tiplied into the Divisor, and the Product wrote under the Dividend, and subtracted out of it. Thus, 3 is in 8 twice, and there remains 2 over. Which Remainder is wont to be placed by the Quotient with the Divisor under it, as thus,  $3)8(2\frac{2}{3}$ . Which  $\frac{2}{3}$  denotes a Fraction as will more fully appear, when we come to speak of common Fractions.

## Chap. 8.

3.  
The first  
particular  
Rule relating to Division.

If the left-hand Figure of the Dividend be greater than the left-hand Figure of the Divisor, then there must be taken at one Time no more Figures (how many soever they be) of the Dividend than are equal to the Number of Figures in the Divisor. And if one Operation being perform'd, what remains of the Dividend, be equal to, or greater than the Divisor, than a second Operation is to be perform'd; and so as many Operations, as there is Occasion. Thus in the adjoining Example, because 8 is bigger than 3, therefore only 8 of the Dividend is to be taken the first Time. Which being divided by 3, gives 2 for the Quotient, and leaves 2 over; which, with the other Figure 7 of the Dividend (plac'd by the Remainder 2) makes

$$\begin{array}{r} 3)87(29 \\ \underline{6} \phantom{0} \\ 27 \phantom{0} \\ \underline{27} \phantom{0} \\ 0 \end{array}$$



ing a Number greater than the Divisor, therefore to be divided anew by the Divisor; upon which Division, there will rise 9 for the Quotient; for  $9 \times 3 = 27$ , and so there will remain nothing of the Dividend given. And consequently 3 is in 87, 29 Times. In like manner, 3 dividing 879, will give for the Quotient 293, as in the Example adjoining.

$$\begin{array}{r} 3 \overline{) 879} \quad (293 \\ \underline{6} \phantom{00} \\ 27 \phantom{0} \\ \underline{27} \phantom{0} \\ 09 \\ \underline{9} \\ 0 \end{array}$$

And here it is to be observed, that so many Figures of the Dividend given as are taken at one Division, are distinguished by the Name of a *Dividual* or *partial Dividend*. And it is usual and convenient to distinguish the several Dividuals of any Dividend, by putting a Point under each Dividual, if of one Figure; or under the last Figure thereof towards the Right-hand, if of more. Thus in the last Example, the Dividend 879 is distinguish'd into three Dividuals of one Figure each, as appears by the Points. And Note, that as many Dividuals or Points as there be in the Dividend, so many Figures must be in the Quotient.

If, when one or more Operations have been perform'd, the remaining Figure (or Figures) of the Dividend, together with the

4.  
The second particular Rule.

5.  
The third particular Rule.

the Remainder (if any there be) of the Operations perform'd, be less than the Divisor, then a Cypher is to be added to the Quotient, and all that remains is to be plac'd by the Quotient, as afore in §. 2. 15) 314 (20  
 Namely, because in the adjoining Example, 15 dividing 31, gives 2 for the Quotient, and leaves 1 over; which Remainder 1, together with the remaining Figure 4 of the Dividend, is less than the Divisor; therefore a Cypher is to be added to the first Quotient 2, and after 2 is to be placed.

The fourth particular Rule.

If, when the Divisor consists of two or more Figures, the said Divisor be compounded, or the Product of several Digits multiplied together, then the Division may be perform'd by the said Digits singly, instead of the Divisor given. Namely, the Dividend given being divided by any one of the single Digits of Divisors first, the Quotient thence arising is to be divided by any other of the said single Digits, and so the Work is to be repeated, so often as there are single Digits. The Quotient arising by the last Operation, is the Quotient sought. Thus 12048 divided by 48; or first singly by 8, and then by 6; (or first by 6, and then

by 8: for  $6 \times 8 = 48$ ,) the Quotient will be found the same, viz. 251, as the following Examples.

1) 12048 (251 or thus, 6) 12048 (2008

$$\begin{array}{r} 96 \\ \hline 244 \\ 240 \\ \hline 48 \\ 48 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 128 \\ \hline 0048 \\ 48 \\ \hline 0 \end{array}$$

and then

8) 2008 (251

$$\begin{array}{r} 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

And the same will hold good, when Divisors do not exactly divide the Dividend, but there is some Remainder: As in the following Examples.

Divid-  
out of  
Divisor's  
Remainder  
Remainder

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48) 12056 (251-7 or thus, 8) 12056 (1507

96

245

240

56

48

800

84

0

0

175) 8002 (8

017

04

04

40

40

056

56

0

and then

6) 1507 (251-7

00

12

30

30

07

6

1

7.  
of Divi-  
on, by  
Naper's  
Rods or  
Bones.

If the Divisor consists of many figures, then the *Division* may with greater Quickness be perform'd by the Help of *Naper's Bones*. Namely, the Divisor being tabulated, by Inspection of the said Bones may easily be perceiv'd, how often the Divisor may be taken out of the respective Dividual. Thus in the following Example, 754 being the Divisor and 2635 the first Dividual, it appears by inspecting the Rods or Bones, that the said Divisor (multiplied by 3, makes 2262, and multiplied by 4, makes 3016 and consequently) is contain'd in its respective



Active Dividual but 3 Times And to the Rest

|   |   |   |   |
|---|---|---|---|
| 1 | 7 | 5 | 4 |
| 2 | 1 | 1 | 8 |
| 3 | 2 | 1 | 5 |
| 4 | 2 | 2 | 6 |
| 5 | 3 | 2 | 2 |
| 6 | 4 | 3 | 4 |
| 7 | 4 | 3 | 2 |
| 8 | 5 | 6 | 3 |
| 9 | 6 | 4 | 6 |

Numbers by  
plain'd in the last Chap.  
the ; together with  
lating to  
There are  
Ways of working  
When the  
I with  
Right-hand of it  
perform'd by  
on the  
there are  
taking the

754) 2635964 (3496

2635  
2739  
3016

7238  
6786

4524  
4524

And (by the Way) this Example, compar'd with the Example of Multiplication, in Chap. 3. §. 10. is an Evidence of

E

8.  
of Divi-  
on, by Lo-  
Carthage.

The first  
particular  
Rule is  
ing to Di-  
vision.

10.  
The sixth  
particular  
Rule.

the

the Proof, of Multiplication by Division  
and of Division by Multiplication.

8.  
Of Division  
on, by Lo-  
garithms.

As for the Method of dividing large Numbers by Logarithms, it shall be plain'd in the last Chapter of this Treatise; together with other Particulars relating to Logarithms.

9.  
The fifth  
particular  
Rule rela-  
ting to Di-  
vision.

There are some other compendious Ways of working Division. Namely when the Divisor consists of the Figure 1, with one or more Cyphers to the Right-hand of it, then the Division perform'd by cutting off so many Figures on the Right-hand of the Dividend, as there are Cyphers in the Divisor, and taking the Rest for the Quotient, the Figures cut off, if significant Ones, belong to be plac'd by the said Quotient, with the Divisor written under them in Manner of a Fraction, as has been afore shewn. For Instance.

10) 100 (10. 10) 12|8 (12. 10) 12|4 (12. 10)

10.  
The sixth  
particular  
Rule.

If the Divisor consists of any other significant Figure than 1, and of Cyphers to the Right-hand, then cutting off so many right-hand Figures of the Dividend, as there be Cyphers in the Divisor, the Division may be perform'd only by the significant Figure (or Figures) of the Divisor, placing (as afore) the Figure

And cut off from the Dividend, as signifi-  
 ed by the Quotient found. Thus,  
 6000) 2148000 (358.

And 2400) 72000 (30.

Where it is to be noted, that the Ex-  
 amples of *Division*, contain'd in this and  
 the fore-going Section of this Chapter,  
 being compar'd with the Examples of  
*Multiplication* contain'd in §. 6. of the  
 fore-going Chapter, will further shew,  
 how *Multiplication* and *Division* mutually  
 prove one the other.

Hitherto I have spoken of the *Division*  
 of Numbers of *one* external Denominati-  
 on. As for the *Division* of Numbers of  
 several external Denominations, if the  
 Divisor be a single Figure, it may easily  
 be perform'd, observing the general Rules.

II.  
 Of dividing  
 Numbers  
 of several  
 external  
 Denomina-  
 tions.

Thus,  $\begin{array}{r} \text{d.} \quad \text{s.} \quad \text{d.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\ 7) 1717 \cdot 71 \cdot 10 \cdot (245 \cdot 6 \cdot 10 \end{array}$

$\begin{array}{r} 14 \\ 81 \\ 28 \\ 87 \\ 36 \end{array}$

2, which 20. being turn'd into  
 £ 2 Shillings,

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Shillings, and the 7 Shillings added to  
to, make a new Dividend to be divid  
by the same Divisor given, 7, viz.

$$\begin{array}{r} 7) \quad 47 \quad (6 \\ \underline{42} \end{array}$$

And this remaining 5 Shillings, be  
turn'd into Pence, and added to the  
Pence given in the Dividend, make  
nother Dividend to be divided by 7,  
given Divisor, viz.

$$\begin{array}{r} 7) \quad 70 \quad (10 \\ \underline{70} \\ 00 \end{array}$$

Whence it appears, that 1717 l. 7 s.  
being divided among seven Persons, e  
Man must have for his Quotient or Sh  
245 l. 6 s. 10 d. And this Example co  
par'd with the Example of *Multiplicati*  
*Chap. 9. §. 12.* further shews, how *M*  
*ultiplication* and *Divison* are mutu  
prov'd one by the other.

12.  
of Divisi-  
on by Re-  
duction.

If the Divisor consists of two or m  
Figures, then the *Divison* may best  
perform'd by *Reduction*, as shall be shew  
*Chap. 10. §. 1. — §. 6.*



It remains only to shew, how *Division* Numbers of one external Denominati- may be prov'd (well enough for com- on Use) by casting away 9: Namely, it be consider'd, that the Dividend in *Division* always answers to the Product in *multiplication*, and the Divisor and Quo- tient here, to the two Factors there. Hence, if, 9 being cast away as often as may be, out of the Divisor and Quotient singly, and after that, out of the Product of the two Remainders first, and, the third Remainder last found, the same with the Remainder of the Dividend, after that 9 is cast away from likewise, as often as may be, then the *Division* is rightly perform'd. Thus,

13.  
Of proving  
Division  
by casting  
away 9.

421) 9683 (23.

421 ... 7. 7x3=35 ... 8.

33 ... 5. 5x3=35 ... 8.

9689 ... 8.

like manner, 3006) 40119078 (8012.

3006 ... 9x2=6.

0013 ... 6.

And 40119078 ... 6.

And thus much for the *Division* of In- or whole Numbers.

## CHAPTER VII.

## Of the Addition, Subtraction, Multiplication, and Division of the Decimal and Sexagesimal Fractions.

1. In Decimal Fractions, the Order of Places is reckon'd from the Left-hand to the Right.

2. Decimal Fractions why so called.

**I**N *Integers*, the Order of the Places has been observ'd, Chap. 2. 8. is reckon'd from the Right-hand to Left; but in decimal Fractions it is reckon'd quite contrary, namely, from Left-hand to the Right. Thus, supposing 20 to be an *Integer*, the 2 stands in the first Place, and 0 in the second; but supposing 20 to be a *Decimal*, the 2 stands in the first Place, and 0 Cypher in the second.

In *Integers*, a Figure signifies ten Times as much in any Place, as it does in the Place next before it in Order, (according to the Table, Chap. 2. §. 23.) but in *Decimals*, a Figure signifies ten Times less in any Place, as it does in the Place next before it in Order. Thus, supposing 220 to be an *Integer*, the Right-hand 2 signifies 2 Tens, the Left-hand 2 signifies 2 Hundreds, or Tens of Tens; but supposing 220 to be a *Decimal*, the Left-hand 2 denotes 2 tenth Parts,

Rig  
ht

that Fraction of this sort are call'd  
 Fractions, namely, because the  
 denominative value, belonging to any  
 place in this sort of Fraction, is a deci-  
 mal tenth Part of the denominative  
 value, belonging to the place next be-  
 low it in Order. Namely, an hundredth  
 Part, is the tenth Part of a tenth Part,  
 a thousandth Part, is the tenth Part  
 of an hundredth Part, &c.  
 The denominative value of decimal  
 Fractions, (as well as of Integers, being  
 noted by their respective Places, hence  
 it is requisite, that those Places in deci-  
 mals (as well as whole) Numbers, which  
 are void of any significant Figure, should  
 nevertheless be filled up with Cyphers,  
 whereby may be known the true  
 Places of the significant Figures. Thus,  
 if I would express two hundredth Parts  
 without any Tenth, I write it thus, 02,  
 where the Cypher fills up the first deci-  
 mal Place, and thereby shews, that the  
 2 stands in the second decimal Place,  
 or the Place of Hundredths. Thus,  
 002, denotes two thousandth Parts,  
 because its denominative Value, being  
 promoted the said Figure into an high-  
 er Place; for Instance, the Decimal 2  
 becomes two Tenth, 0.2

Excepting  
 the Diff-  
 erence of  
 the diff-  
 erence of  
 the Or-  
 der of  
 Places in  
 Decimals  
 and Int-  
 eger

8. The Use  
 of Cy-  
 phers in  
 Decimals;  
 the same  
 as in whole  
 Numbers.

4.

Excepting  
the Difference  
arising from  
the different  
Ways  
of reckon-  
ing the Or-  
der of  
Places in  
Decimals  
and Inte-  
gers.

But now, although the Use of Cyphers in Decimals is the same (consider'd in itself) as in *Integers*, namely, to fill vacant Places, and so to shew the true Places of the significant Figures; there is this Difference arising from the two different Ways of reckoning the Order of Places in Decimals, and in *Integers*. Namely, a Cypher set on the Right-hand of an integral Figure, promotes or increases (as has been observ'd Chap. 1. §. 30.) its denominative Value, because it promotes the said Figure into an higher Place; for Instance, 2 *Twenty*, 200 *two Hundred*, &c. But a Cypher set on the Right-hand of a decimal Figure, does not promote its denominative Value, because it does not promote the said Figure into an higher Place; for Instance, 2 (supposing a Decimal, when alone) denotes *two Tenths*, and the decimal 20 denotes likewise the same, as also the decimal 200 denotes the same: because the said 2 in each of the three Instances stands in the same, viz. the first decimal Place. Whereas, on the contrary, a Cypher set on the Left-hand of a decimal Figure, does promote its denominative Value, because it promotes the said Figure into an higher Place; for Instance, the Decimal 2 denotes *two Tenths*, p2 *two Hundredth*, o





*Thousandth Parts, &c.* But a *0* after  
 the Left-hand of an Integral Figure,  
 does not promote its denominative Va-  
 lue, because it does not promote the said  
 Figure into an higher Place: for In-  
 stance, the *Integer* 2, or 02, or 002, does  
 each denote no more than two Units.  
 It is next to be observ'd, that (where-  
 the Places of *Integers*, are reckon'd  
 from the Place of Units *inclusively*) the  
 Places of decimal Numbers, are reckon'd  
 from the Place of Units *exclusively*. Hence  
 is the Denomination of the second  
 place in *Integers*, but *Tenths* of the first  
 place in Decimals, &c. And it is usual  
 and necessary to denote, whether the  
 Number given be an *Integer* or a Deci-  
 mal, by putting some Mark before it,  
 viz. either a full Point, or a Commas, or  
 this mark (*l*). And by the same Marks  
 the decimal Figures are distinguish'd from  
 the integral in Numbers given, and con-  
 sisting partly of Integral, and partly of  
 decimal Figures. Thus .2 or ,2 or l2 de-  
 note two *Tenths*; .03 three *Hundredths*;  
 .004 four *Thousandths*; 2.43, or 2,l43,  
 or 2.43, denote two *Units* (or *Wholes*) four  
*Tenths*, and three *Hundredth Parts* (or for  
 three *Hundredth Parts*) of one such  
 Whole.

9.  
 Decimal  
 Numbers  
 how di-  
 stinguish'd  
 from Inte-  
 gers, &c.

Lastly,

The Agree-  
ment be-  
tween  
working  
with Inte-  
gers, and  
working  
with De-  
cimals,  
whence  
it arises.

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*Left.* It is to be observ'd, that though the Places of *Integers* are reckon'd in a different Manner (as has been observ'd) from the Places of *Decimals* as to their Order, namely, the Places of *Integers* from the Right-hand, the Places of *Decimals* from the left; yet there is an exact Agreement between them in this, viz. that the denominative Value of every left-hand Place, is ten Times as much as the denominative Value of the next fore-going right-hand Place. Namely, in decimal Numbers, *ten* Thousandths Parts make *one* Hundredth Part, and *ten* Hundredth Parts make *one* Tenth Part, and *ten* Tenth Parts make *one* Unit or Whole; and so on likewise in *Integers*, *ten* Units make *one* Ten, *ten* Tens make *one* Hundred, *ten* Hundreds make *one* Thousand, &c. And on this Agreement as to the Increase of the denominative Value of *Integers* and *Decimals*, depends and is founded the Agreement between *Integers* and *Decimals* as to the Manner of Working, i. e. as to *Addition*, *Subtraction*, *Multiplication*, *Division*, &c. For these Operations are perform'd in *Decimals*, just as they are in *Integers*, according to the general Rules above laid down, *Chap. II.* The only Particular, that is further requisite to be known as peculiarly belonging to the Working

Decimals, and to distinguish them from the Integral  
 Multiplication and Division. Which  
 therefore shall be shewn in the proper  
 places. Addition of Decimals.

**EXAMPLE I.** Addition of Decimals.

$$\begin{array}{r} 25 \\ 0578 \\ 000 \\ 8 \\ \hline 25.0578 \end{array}$$

**EXAMPLE II.** Addition of Decimals.  
 Of Integers and Decimals together.

$$\begin{array}{r} 745.25 \\ 63.054 \\ 1000.0008 \\ 278.918 \\ \hline 2091.7208 \end{array}$$

$$\begin{array}{r} 25 \\ 0578 \\ 000 \\ 8 \\ \hline 25.0578 \end{array}$$

$$\begin{array}{r} 745.25 \\ 63.054 \\ 1000.0008 \\ 278.918 \\ \hline 2091.7208 \end{array}$$

By the fore-going Examples it evident-  
 ly appears, that in *Addition* the Deci-  
 mals are rightly separated from the Inte-  
 gers, by working according to the first  
 and second general Rule laid down,  
 Chap. 2. §. 8 & 9. And the same will  
 appear further from the following Ex-  
 amples of *Subtraction*. Which Exam-  
 ples, compar'd with the fore-going Ex-  
 amples of *Addition*, will also shew, that  
*Addition* and *Subtraction* do mutually  
 prove

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prove each other in Decimals, as well as Integers.

8:  
Subtraction  
of Decimals

## Subtraction of Decimals.

EXAMPLE I, EXAMPLE II

$$\begin{array}{r} 1.7203 \\ .6034 \\ \hline 1.1168 \end{array}$$

$$\begin{array}{r} 3091.7208 \\ 278.916 \\ \hline 1812.8048 \end{array}$$

9:  
Multiplication  
of Decimals

## Multiplication of Decimals.

EXAMPLE I, EXAMPLE II  
Of Decimals alone, Of Decimals alone

$$\begin{array}{r} .904 \\ .68 \\ \hline 7232 \\ 5424 \\ \hline .61472 \end{array}$$

$$\begin{array}{r} .7089 \\ .034 \\ \hline 28356 \\ 21867 \\ \hline .0241026 \end{array}$$



### EXAMPLE III,

Of Integers and Decimals together.

$$\begin{array}{r}
 74.05 \\
 5.924 \\
 \hline
 29620 \\
 124810 \\
 \hline
 66645 \\
 22215 \\
 \hline
 290.57220
 \end{array}$$

### EXAMPLE IV,

Of Integers and Decimals together.

$$\begin{array}{r}
 2.45 \\
 1.0008 \\
 \hline
 1960 \\
 245000 \\
 \hline
 2.451960
 \end{array}$$

N. B. In working the particular Products, Regard need to be had to the placing of the Figures of the said Products no further, than that the first Figure of each Product stands one Place more to the Left-hand, than the first Figure of the former Product; and so the Rest in Order. And the total Product being found by the Addition of the particular Products, the said total Product must have as many decimal Figures, as there are in both the Factors together. Which is exemplify'd in all the fore-going Examples. What Figures there are in the said Product over the said Number, they are Integers, as in Example 3d and 4th. If the said total Product has not so many Figures

gures in all, as there be Decimals in the Factors, then Cyphers must be added to the Left-hand of the Figures of the Product, till they become equal in Number to the Decimals in the Factors, Example the second.

10.  
Division of  
Decimals.

Division of Decimals.

EXAMPLE I,      EXAMPLE II  
Of Decimals alone.      Of Decimals alone.

|                   |                            |
|-------------------|----------------------------|
| 168) .61471 (.904 | 17089) .0841086 (.94 CH 4) |
| 618               | 8167                       |
| <u>618</u>        | <u>8167</u>                |
| 000               | 0000                       |
| 471               | 0000                       |
| <u>471</u>        | <u>0000</u>                |
| 000               | 0000                       |

EXAMPLE III,      EXAMPLE IV  
Of Integers and Decimals together.      Of Integers and Decimals together.

|                         |                      |
|-------------------------|----------------------|
| 74.09) 290.37220 (3.924 | 2.49) 2.431980 (1.00 |
| 2225                    | 249                  |
| <u>2225</u>             | <u>249</u>           |
| 6822                    | 61980                |
| <u>6822</u>             | <u>61980</u>         |
| 27772                   | 27720                |
| <u>27772</u>            | <u>27720</u>         |
| 520                     | 520                  |
| <u>520</u>              | <u>520</u>           |
| 000                     | 000                  |

As by the fore-going Examples of *Division*, compar'd with the fore-going Examples of *Multiplication*, it is evidently shewn, how *Multiplication* and *Division* mutually prove each other, in Decimals, as well as *Integers*; so also it is from hence evidently shewn, that (as in *Multiplication*, the Product must contain as many Decimals, as are given in the two Factors together, so) in *Division*, the Divisor and Quotient must contain together as many Decimals, as are given in the Dividend. Which is exemplify'd in the fore-going Examples of *Division*. When so many Figures are set off for Decimals in the Quotient, as is requisite together with the Decimals given in the Divisor, (if any be given) to equal the Number of Decimals given in the Dividend, the remaining left-hand Figures of the Quotient are *Integers*, as in Example III, and IV. But if the Figures arising in the Quotient according to the common Method of *Division*, will not all of them together with the Decimals given in the Divisor equal the Number of Decimals given in the Dividend, then one or moreiphers (as is requisite) must be prefixt to the Figures already found in the Quotient; as in Example II, where the Quotient found according to the common Method of *Division*, is 34; which, because

craft together with the four Figs  
the Divisor, they will not make a  
Number of Decimals given in the D  
dend, viz. seven, therefore the Quo  
is to be made 054, and all the three  
gures are to be esteem'd Decimals.

## II. Of turning common Fractions into Deci- mal.

How *common* Fractions may be  
duc'd or turn'd into *decimal* Fraction  
shewn, *Chap. 10. §. 13.* Proceed  
now to speak of *common* Fractions.



## CHAP. VIII.

The Addition, Subtraction, Multiplication, and Division, of vulgar or common Fractions.

**1.** **Vulgar or common Fractions** are so call'd, because they were vulgarly commonly made Use of, before that decimal Fractions were invented, and were known.

**Common Fractions, why so call'd.**

The great Difference between common and decimal Fractions lies in this, that the denominative Values, or in other words, the Denominators of decimal Fractions (like as of Integers) are known by the Places of the decimal Figures, and proceed regularly in a like Proportion; whereas the Denominators of common Fractions are not to be known by the Places of their Figures, forasmuch as they proceed not in any regular uniform Proportion.

In the Notation of common Fractions it is requisite, to write down not only their numerative Values or Numerators, but also their denominative

Values

**2.** The great Difference between Decimal and common Fractions, wherein it lies.

**3.** The Denominators of common Fractions are to be writ down.

Values or Denominators. For Instance 2 tenth Parts are by a common Fraction denoted usually thus,  $\frac{2}{10}$ ; whereas the same is express'd by a decimal Fraction thus, .2, namely, (as has been observ'd,) the Place of the Decimal standing known to be the first Place of Decimals, thereby it is known also, that said 2 denotes 2 Tenths. In like manner, 2 hundredth Parts are express'd by a common Fraction thus,  $\frac{2}{100}$ , and a thousandth Parts thus,  $\frac{2}{1000}$ , whereas the former is express'd as a decimal Fraction thus, .02, and the latter thus, .002. Namely, the decimal Figure 2 standing in the former Fraction in the second Place, is known thereby to denote hundredth Parts; and in the latter Fraction standing in the third Place, is known thereby to denote a thousandth Part. Hence, what is express'd by the common Fractions  $\frac{2}{10}$ , and  $\frac{2}{100}$ , and  $\frac{2}{1000}$ , is express'd much shorter by the Decimals .2, .02, and .002. And this shorter Way of Notation by Decimals, than by common Fractions is one Reason which has made the Use of decimal Fractions be so generally prefer'd to the Use of common Fractions.

But that, wherein decimal Fractions chiefly excell common Fractions; is this; that decimal Fractions (by reason of their Denominators proceeding regularly in the like Proportion, and so being denoted by their Places, as are the Denominators of Integers) are work'd after the same Manner as Integers; whereas common Fractions can't be so work'd in most Cases, (namely, as often, and they have different Denominators, which proceed not regularly in a like Proportion); but must be first reduced to one and the same common Denominator. As often as common Fractions have the same Denominators, they may be writ as whole Numbers, and so work'd in no Manner of Difficulty; as shall be shewn in the following Examples. Which may be of Use first to oblige here, that, (whereas the common Way of writing common Fractions, is to place the denominator under the Numerator with a Line between, which occasions some seeming Difference between the Operations of Integers, and common Fractions of the same Denomination; in order to take away even this seeming Difference,) it is very advisable to write common Fractions, as we write Numbers of several external Denominations, that is, to

4.  
The Excellency of decimal Fractions above Common, wherein it chiefly lies.

5.  
Common Fractions of the same Denomination, may be work'd as Integers; especially, if writ after the Method here shewn.

4 place the Denominators at the Head  
 Top of the Numerators of their re-  
 spective common Fractions. Namely,  
 we denote two Shillings and six Pence  
 thus,  $2\frac{6}{12}$  and  $6\frac{6}{12}$ , so it seems better to  
 note the same by common Fractions  
 thus,  $2\frac{1}{2}$  and  $6\frac{1}{2}$ , than thus (as usu-  
 ally)  $2\frac{3}{6}$  and  $6\frac{3}{6}$ . For the former Way  
 ing the same, whereby Integers of seve-  
 ral external Denominations are denoted  
 hence common Fractions being express'd  
 after the same Way, the Working of the  
 will more appear to be agreeable to the  
 Working of these, than if the common  
 Fractions were express'd the common  
 Way. However, to comply with Cu-  
 stom, the common Way of Expressing  
 and Working common Fractions shall  
 be altogether here omitted, but plac'd  
 the Side of the other Way.

2 Common Fractions of the same Denomination may be work'd as Integers; especially, if we use the Method here perform-  
 ing Difference between the Quotient and common Fractions of Integers, and common Fractions of Integers; in order to  
 away even this seeming Difference, it is very advisable to write common  
 Fractions as we write Numbers of seve-  
 ral external Denominations, that is to  
 place



# **Addition of common Fractions, of the same Denomination.**

68  
Addition  
of common  
Fractions  
of the same  
Denomina-  
tion.

## **EXAMPLE I.**

## **EXAMPLE II.**

For in both Cases the Operation  
being of like Nature, namely by multiplying  
the Numerators given one into the other  
and also the Denominators, for

$$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

$$\frac{3}{6} + \frac{5}{6} = \frac{8}{6}$$

$$\frac{2}{20} + \frac{6}{20} = \frac{8}{20}$$

$$\frac{1}{20} + \frac{6}{20} = \frac{7}{20}$$

# **Subtraction of common Fractions, of the same Denomination.**

7.  
Subtracti-  
on of the  
same.

## **EXAMPLE I.**

## **EXAMPLE II.**

$$11 - 8 = 3$$

$$13 - 6 = 7$$

$$\frac{8}{12} - \frac{5}{12} = \frac{3}{12}$$

$$\frac{6}{20} - \frac{1}{20} = \frac{5}{20}$$

In this Way of Working and Working, Learners are  
to enquire, why only the Numerators are added and  
abstracted, and not also the Denominators. Whereas  
the Reason thereof is made evident by the other Way,  
which shews the Manner of Working to be the same as in  
the Case of several external Denominations.

F 3

Multi-

2.  
Multiplication of  
common  
Fractions  
in general

**Multiplication of common Fractions  
of the same, and also of different  
Denominations.**

For in both Cases the Operation  
perform'd alike, namely, by multiplying  
the Numerators given one into the other  
and also the Denominators. For In-  
stance :

### EXAMPLE I.

$$3^4 \times 2^4 = 6^4 \text{ or } 8^4 \div 4 = 16$$

### EXAMPLE II.

$$3^4 \times 2^4 = 6^4 \text{ or } 8^4 \div 4 = 16$$

Nor is the *Multiplication* of common Fra-  
ctions as to their Denominators (as was  
done in such *Integers* as are capable of  
such *Multiplication*, namely, *Integers*  
Measure, as Yards, Feet, &c. For when  
2 Feet are multiplied into 2 Feet, not only  
the Numerators 2 and 2 are multiplied  
one into the other, but also the Denomi-  
nators Feet, (viz. in Length) are multi-  
plied one into the other, and produce  
Feet Square. Namely, as  $3^4 \times 2^4 = 6^4$   
so  $3^4 \times 2^4 = 6^4$  sq. i. e. 16.

Divi

Division of common Fractions, of the same Denomination.

9. Division of common Fractions, of the same Denomination.

performed by dividing only the Numerator of the Dividend given, by the Numerator of the given Divisor. For instance :

$24 \div 6 = (3 \text{ or } \frac{24}{6}) = 4$

If the common Fractions given to be added, or subtracted, or divided, be of different Denominations, then they must (as has been already observ'd) reduced to one and the same Denomination. And now this is to be done, shall be shewn in Chap. 10. Sect. 7.

It remains here only to observe, that a Fraction being a Part of some one thing, therefore its Numerator ought properly to be less than its Denominator; as  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \text{ \&c.}$  Which are therefore call'd proper common Fractions.

If the Numerator of a common Fraction be equal to, or greater than the Denominator, then it is call'd an improper fraction, because, the Quantity denoted thereby is, either equal to one Whole, as in the first Case; or else greater than one Whole, as in the second Case. Thus,  $\frac{2}{2} = 1, \frac{3}{2} = 1\frac{1}{2}, \frac{4}{2} = 2, \frac{5}{2} = 2\frac{1}{2}, \text{ \&c.}$

Namely, how much an improper Fraction is greater than One, is known by dividing the Numerator by the Denominator. For as often as the Denominator is contain'd in the Numerator, so many Wholes are denoted by the improper Fraction, and what is over, is a proper Fraction, or another Whole.

12.

A mixt Fraction, what.

A whole Number with a common Fraction by it, is call'd a mixt Fraction, as  $1\frac{1}{2}$ ,  $2\frac{1}{4}$ ,  $17\frac{3}{4}$ , &c.

13.

A Fraction of a Fraction, what.

When a Fraction denotes a Part of another Fraction, then it is call'd, a Fraction of a Fraction, or a compound Fraction. Thus,  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$  of a Shilling is a Fraction of a Fraction, and denotes three Pence.

14.

Of the Reduction of the several Sorts of Fractions, one into the other.

How these several Sorts of common Fractions, may be reduc'd one into another, (for which there is frequently Occasion,) is shewn in Chap. 10.

15.

Of the Value denoted by common Fractions.

The last Thing to be here observ'd, and which is to be taken special Notice of, as being of great Use, is this, that in common Fractions what is principally to be regarded, is the Proportion of the Numerator to the Denominator. For if this be the same, the Fractions are equivalent, how much soever the Numbers, whereby they are express'd, may be different. Thus,  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$ , &c. Wherefore the Wo



may frequently be shorten'd, by taking an Equivalent Fraction instead of a greater. And how any common Fraction given may be reduc'd to its least equivalent Fraction, is also shewn in Chap. §. 11. And so much may suffice concerning common Fractions in this place.

As for Sexagesimal Fractions, there is no Manner of Difficulty as to working with them, it being exactly the same as that of Integers of several external Denominations. For indeed such Integers may also esteem'd Fractions, viz. Shillings may be esteem'd and call'd *Vigesimal Fractions*, forasmuch as every Shilling is the twentieth Part of a Pound Sterling; and Pence may be call'd *Duodecimal Fractions*, forasmuch as every Penny is the twelfth Part of a Shilling; and so of other Parts of Money. And the same holds good as to the several Denominations of Measure; as also, as to Days and Hours in Time. But it is usual to look upon these, for Instance, Days and Hours, rather as whole Things, than as Fractions, viz. Days of Years or Months, or Years and Hours of Days. Whereas, on the contrary, it is usual to look on Minutes and Seconds of Time, not as Wholes, but as Fractions or Parts, viz. Minutes

16.  
Of Sexagesimal  
Fractions.

Minutes of Hours, and Seconds of Minutes; and therefore to call them Sexagesimal Fractions, namely, because a Minute is the sixtieth Part of an Hour; a Second the sixtieth Part of a Minute. And the like is to be understood, as Minutes, Seconds, Thirds, Fourths, &c. in local Extent. For as such a Minute esteem'd the sixtieth Part of a Degree, such a Second is esteem'd the sixtieth Part of such a Minute, a Third the sixtieth Part of a Second, a Fourth of a Third, &c. Hence it will be sufficient to add here an Example or Two in Addition and Subtraction.

17.  
Addition  
of Sexagesimal Fractions.

Addition of Sexagesimal Fractions.

EXAMPLE I.

EXAMPLE II.

| Hours. | Minutes. | Seconds. | Degrees. | Minutes. | Seconds. |
|--------|----------|----------|----------|----------|----------|
| 20.    | 43.      | 29.      | 125.     | 17.      | 00.      |
| 00.    | 50.      | 41.      | 36.      | 52.      | 16.      |
|        |          |          | 00.      | 50.      | 49.      |
| 21.    | 34.      | 10.      | 162.     | 10.      | 06.      |

# Subtraction of Sexagesimal Fracti-

18.

Substrac-  
tion of Sexa-  
gesimal  
Fractions.

EXAMPLE I.

EXAMPLE II.

|    |    | Degrees. |    |     |    |
|----|----|----------|----|-----|----|
| I  | II | I        | II | III | IV |
| 34 | 10 | 162      | 10 | 06  | 18 |
| 50 | 41 | 125      | 17 | 00  | 59 |
| 43 | 29 | 36       | 53 | 05  | 18 |
|    |    |          |    |     | 39 |

The Great  
Difference  
between  
Algebra  
and com-  
mon Arith-  
metick  
wherein  
it lies.

No sign-  
fiance  
and nega-  
tive Quan-  
ties what.

## CHAP.

## C H A P. IX.

## Of Algebraical Addition, Subtraction, Multiplication, and Division.

1.  
The great  
Difference  
between  
Algebra  
and com-  
mon Arith-  
metick,  
wherein  
it lies.

**I**N (\*) Algebraical Operation's Letters are made Use of to denote, either Numbers themselves, or the Things number'd. The great Difference Between Algebraical and common Arithmetick lies in this, that *Algebra* admits of negative Quantities, which common Arithmetick does not.

2.  
An affirmative  
and negative Quantity, what.

Where it is to be noted, that, as an affirmative Quantity, is denoted for a real Thing it self; so by a negative Quantity, is denoted the Defect or Want of some real Thing. An affirmative Quantity, either actually has, or else is suppos'd to have the Sign  $+$  affixt to the Left-hand of it; and a negative Quantity, always has the Sign  $-$  actually affixt to the Left on it. Inſomuch, that a

(\*) *Algebra* denotes in the *Arabick* Language, as much as the Great or Excellent Art.

(†) Namely, it is usual, not actually to affix the  $+$  to a single affirmative Quantity, or to an affirmative Quantity, that stands at the Beginning of any compound Quantity.

Quant



quantity, which has no Sign to affix  
it, is to be esteem'd an Affirmative.  
us,  $3A$  (or  $3B$ , or  $3C$ , &c.) may de-  
note three Shillings, and then  $-3A$  (or  
 $-3B$ , or  $-3C$ , &c.) will denote the  
want of three Shillings. Now, that the  
primary Operations in *Algebra*, are  
agreeable to those in common Arithme-  
tick, but where the Nature of negative  
quantities cause some Variation, will  
appear as we go along the said Opera-  
tions.

### Algebraical Addition

Others into one Sum Quantities, of  
the same Denomination, i. e. Quantities  
which are express'd by the same Letter or  
Letters, and by the same Sign, Affirma-  
tive or Negative. For Instance:

The Addition  
of  
Quantities  
of the same  
Denomina-  
tion.

#### EXAMPLE I.

|       |                     |       |
|-------|---------------------|-------|
| $2A$  | Or more agreeable   | $2$   |
| $3A$  | to the Notation of  | $5$   |
| <hr/> |                     | <hr/> |
| $7A$  | common Arithmetick. | $7$   |

#### EXAMPLE II. EXAMPLE III.

|                |             |                 |
|----------------|-------------|-----------------|
| $-A$ or $-1$ . | $A + B - C$ | Cor $1 + 1 - 1$ |
| $-6A$          | $-6$ .      | $4A + 3B - 5C$  |
| <hr/>          | <hr/>       | <hr/>           |
| $-7A$          | $-7$ .      | $5A + 3B - 6C$  |
|                |             | $5 + 3 - 6$     |
|                |             | Namely,         |

Namely, as in Example I, 2 Shillings (denoted by 2A) added to 5 Shillings (denoted by 5A) make together 7 Shillings (denoted by 7A), so in Example II, the Want of 6 Shillings (denoted by —6A) added to the Want of one Shilling (denoted by —A) makes together the Want of 7 Shillings, denoted by —7A.

2. If the Quantities be of several Denominations, then, there are two different Ways of Working, according to two different Cases that may happen. Namely,

3. Case 1<sup>st</sup>, If the Species or Letters, which the Quantities are exprest, be different, then they can't be properly added, i. e. can't be gather'd into one Sum, but can only be connected together with the Sign of Addition, (viz. +,) either exprest or understood. Thus,

## EXAMPLE IV.

$$\begin{array}{r} A \quad \text{or} \quad 1A \\ B \quad \text{or} \quad 1B \\ \hline A+B \quad \text{or} \quad 1A+1B \end{array}$$

## EXAMPLE V.

$$\begin{array}{r} -A \\ 3C \\ \hline -A+3C \quad \text{or} \quad 3C-A \end{array}$$

## EXAMPLE VI.

$$\begin{array}{r} 2B \\ 5C \\ \hline 2B+5C \quad \text{or} \quad 5C+2B \end{array}$$

here it is to be noted, that in this Algebra is agreeable to common Arithmetic. For therein likewise Numbers of several Denominations can't be properly added, while they continue such, but only connected by the Sign of Addition, which it is usual to understand, in order to express. Thus 2 Shillings and 3 Pence can't be added together, while they continue in the said different Denominations, but by writing them together, as, 2. 3, that is, 2 + 3. Just as A and 3B, are added together in Algebra, thus,  $2A + 3B$ .

Case 2d, If the Letters be the same, but the Signs of the given Quantities be different, then the said Quantities destroy each the other in a like Proportion or Number.

EXAMPLE VII. EXAMPLE VIII. EXAMP. IX.

|                            |                            |                            |
|----------------------------|----------------------------|----------------------------|
| 5A                         | -6B                        | 7BC                        |
| -3A                        | 4B                         | -BC                        |
| <hr style="width: 100%;"/> | <hr style="width: 100%;"/> | <hr style="width: 100%;"/> |
| 2A                         | -2B                        | 6BC                        |

The Reason of thus Working will be evident, if the Nature of negative Quantities be consider'd. For a negative Quantity denoting the Defect of a Thing, is in effect to add a negative Quantity, is in effect to subtract. Hence, in Example VII,

VII. to add  $-3A$  to  $5A$  is the same to subtract  $3A$  from  $5A$ . And there from such an *Addition*, there arises only  $2A$  for the Sum, which is really Remainder of the greater Quantity above the less. And so in the other examples.

5.  
Subtracti-  
on of  
Quantities  
of the same  
Denomina-  
tion.

### Algebraical Subtraction

takes the less given Quantity out of the greater, if they be both of the same nomination.

#### EXAMPLE I. EXAMPLE

$$\begin{array}{r} 7A \text{ or } 7 \\ 3A \quad 3 \\ \hline 4A \quad 4 \end{array}$$

$$\begin{array}{r} -7A \text{ or } -7 \\ -A \quad -1 \\ \hline -6A \quad -6 \end{array}$$

#### EXAMPLE II.

$$\begin{array}{r} 5A + 3B - 6C \text{ or } 5 + 3 - 6 \\ 4A + 2B - 5C \quad 4 + 2 - 5 \\ \hline A + B - C \quad 1 + 1 - 1 \end{array}$$



If the Quantities be of *several* Denominations, then there are two different ways of Working, (as afore in *Addition*.) according to the two different Cases, that may happen. Namely,  
*Case 1<sup>st</sup>*, If the Letters be different, then the Quantities express thereby can't properly be subtracted, but it can only be shewn, by the Sign of *Subtraction*, (i. e. —) plac'd between the Quantities, that they are to be subtracted one from the other.

6. Subtraction of Quantities of several Denominations.

7. *Case the 1<sup>st</sup>*.

EXAMPLE IV. EXAMPLE V. EXAMPLE VI.

|       |       |               |
|-------|-------|---------------|
| A     | —A    | —2B           |
| B     | 3C    | —5C           |
| <hr/> |       |               |
| A—B   | —A—3C | —2B(—)—5C     |
|       |       | i. e. —2B+5C. |

As to Example IV and V, it is to be noted, that the Algebraical Operation does therein to the common *Subtraction*. For therein likewise Numbers of several Denominations can't be properly subtracted, while they continue such, from the other. For nine Pence can't be subtracted from twelve Shillings, while they all continue Shillings; it can only be shewn, that the nine Pence are to be subtracted from the twelve

G

twelve Shillings, by writing them

$$12 - 9.$$

And as to Example VI, it is evident that to *subtract a Negative*, is really *add*. Wherefore to subtract  $-5C$  to add  $5C$ .

8.  
Case the  
2d.

And this last Consideration shews Reason of the Operation in the second Case, namely, when the Species or Letters be the same, but the Signs differ. Namely,

EXAMPLE VII. EXAMPLE VIII. EXAMPLE IX.

$$\begin{array}{r} 2A \\ -3A \\ \hline 5A \end{array}$$

$$\begin{array}{r} -2B \\ 4B \\ \hline -6B \end{array}$$

$$\begin{array}{r} 6BC \\ -BC \\ \hline 7BC \end{array}$$

For in Example VII, to subtract  $-3A$  is to add  $3A$ , and consequently, the Quantity found must be  $5A$ . And for Example IX, to subtract  $-BC$ , is to add  $BC$ , and therefore the Quantity arising from such a Substraction, must be  $7BC$ . On the contrary, in Example VIII, to subtract  $4B$ , being the same as to add  $-4B$ , therefore the Quantity found must be  $-6B$ .

9.  
Algebraical Addition and Substraction.

Lastly, It is to be observ'd, that comparing the several fore-going Examples one with the other, (*viz.* Exam

*Arithmetick*, &c. I  
of Addition, with Example iv. of Sub-  
traction, &c.) it will thence appear, that  
Addition and Subtraction do likewise mu-  
tually prove one the other, in Algebra-  
ical, as well as common Arithmetick.

on, mutu-  
ally prove  
one the o-  
ther.

### Algebraical Multiplication

10.  
of Multi-  
plication

multiplies the Quantities given one into  
the other, whether they be of the same  
different Denominations, namely, it  
multiplies Numerator into Numerator,  
and Denominator into Denominator:

#### EXAMPLE I.

BA      a      a      aa Denominators  
BA    or 3 \* 2 = 6. Numerators:  
AA

#### EXAMPLE II.

BC      Denominator.    b      c      bc  
or  
BC      Numerator.      5 \* 3 = 15.

And here it is to be noted, that any  
series of Letter, being multiplied into  
itself once, its Product is express'd these  
several

113  
Of the  
Manner of  
expressing  
the Pro-

ducts of  
the same  
Letter.

# The Young Gentleman's

several Ways, viz.  $A \cdot A = AA$ , or  $A^2$ , or  $AA$ , i. e. A Square. In like manner,  $AAA$  (or  $AA \cdot A$ ) =  $AAA$ , or  $A^3$ , or  $AAA$ , i. e. A cubed. And so on, of which more, Chap. 12.

12.  
Of the  
Signs to be  
affix'd to  
the Pro-  
ducts.

It is also to be noted, that, if the Signs be alike, then the Product will always Affirmative; but if the Signs be unlike, then the Product will be always Negative. For the Multiplier, if be Affirmative, puts down; if Negative takes away the Multiplicand, whether this be Affirmative or Negative. Wherefore,

|                    |                           |                                 |                                                     |                    |
|--------------------|---------------------------|---------------------------------|-----------------------------------------------------|--------------------|
| $+$ into $+$       | } gives $+$ ,<br>that is, | } A Thing so often put<br>down, | } or,<br>The Want thereof so of-<br>ten taken away. | } come<br>the same |
| or<br>$-$ into $-$ |                           |                                 |                                                     |                    |

In like manner,

|              |                           |                                    |                                                             |                    |
|--------------|---------------------------|------------------------------------|-------------------------------------------------------------|--------------------|
| $+$ into $-$ | } gives $-$ ,<br>that is, | } A Thing so often took a-<br>way, | } or,<br>The Want thereof so of-<br>ten put down or caus'd. | } come<br>the same |
| $-$ into $+$ |                           |                                    |                                                             |                    |

(III) N. B.  $A \cdot A = AA$ , but  $2A = A + A$ . So  $A^2 = AA$   
but  $2A = A + A$ .

Examp



Example III. Example IV. Example V. Example VII.

$$\begin{array}{r} A \\ B \\ \hline AB \end{array} \quad \begin{array}{r} 3B \\ 3B \\ \hline 6B \end{array} \quad \begin{array}{r} 5C \\ 5C \\ \hline 10C \end{array} \quad \begin{array}{r} 4D \\ 3D \\ \hline 12DE \end{array}$$

In compound Quantities, the Operation may proceed from Right to Left, as in common Arithmetick; but it is more usual to perform Algebraical Multiplication from Left Right. Thus,

### EXAMPLE VI.

|             |               |                       |                  |
|-------------|---------------|-----------------------|------------------|
| Pro. of A.  | $A + E$       | So likewise may com-  | $10 + 2$         |
| into A      | $A + E$       | mon Multiplication be | $10 + 2$         |
| Pro. of E.  | $A + E$       | perform'd, viz.       |                  |
| into A      | $A + E$       |                       | $100 + 20$       |
| total Prod. | $A + 2AE + E$ |                       | $+ 20 + 4$       |
|             |               |                       | $100 + 40 + 4$   |
|             |               |                       | that is in short |
|             |               |                       | 144.             |

### EXAMPLE VII.

|             |               |                |                       |
|-------------|---------------|----------------|-----------------------|
| Pro. of A.  | $A + 2AE + E$ | So in common   | $100 + 40 + 4$        |
| Pro. of E.  | $A + E$       | Multiplication | $10 + 2$              |
| total Prod. | $A + 2AE + E$ |                | $1000 + 400 + 40$     |
|             |               |                | $+ 200 + 80 + 8$      |
|             |               | Way, as on     | $1000 + 700 + 20 + 8$ |
|             |               | the side       | that is in short      |
|             |               |                | 1728.                 |

N. B. A thorough Insight into these two Examples, is of great Use, as tending to render

render the Extraction of the square  
cube Root easy to be apprehended,  
consequently performed. Of which  
more, Chap. 12.

14.  
of Divi-  
son.

## Algebraical Division

takes the Divisor, as often as may  
out of the Dividend, and places the  
remainder in the Quotient. Where it is  
be noted, that Numerator is to be di-  
ded by Numerator, and Denominator  
Denominator.

### EXAMPLE I.

Denominator a  
3A) 6AA (2A or, Numerator 2) 6

### EXAMPLE II.

Denominator c  
3C) 15BC (5B or, Numerator 3) 15

In like manner, A)AA(A, or A)A'(A,  
A)Aq(A, Also A)AAA(AA, or A)A'(A'  
A)Ac(Aq. Also AA)AAA(A, or A') A'  
or Aq)Ac(A, &c.

15.  
of the Sign  
to be affect  
to the  
Quotient.

If the Signs given be alike, then  
Quantity found by Division will be A-  
mative; if the Signs be unlike, then  
Quantity found will be Negative. F  
whereas Division undoes, what Multi

V+E)

Y E R  
C u  
d i  
o r

of

9



DC

2

2001

**Q**

old

## EXAMPLE VIII.

$$A+E) A^2+3A^2E+3AE^2+E^3 \quad (A^2+2AE+E^2) \\ A^2+2A^2E+AE^2$$

$$\begin{array}{r} 0 \quad A^2E+2AE^2 \\ A^2E+2AE^2+E^3 \\ \hline 0 \end{array}$$

16.  
Algebraical Multi-  
plication  
and Divi-  
sion, mutu-  
ally prove  
one the  
other.

17.  
The best  
Way to  
perform  
Algebrai-  
cal Divi-  
sion.

18.  
Of the Divi-  
sion of  
Quantities, which  
are of Denomina-  
tions altogether Different.

By comparing the fore-going Examples of *Division*, with the correspondent Examples of *Multiplication*, will appear, that Algebraical (as well as common) *Multiplication* and *Division*, do mutually prove one the other.

And also it will appear, that the best Way to perform Algebraical *Division*, is by considering, what is the Quantity, which being multiplied by the Divisor, will produce the Dividend. For that is always the Quotient.

Lastly, If the Divisor and Dividend be altogether of different Denominations then the *Division* can be no otherwise perform'd, than by shewing, that the said Quantities are to be divided, which is done by writing them like a Fraction.

Thus,  $A) B \left( \frac{B}{A} \right)$

And so much for the four principal Operations in *Algebra*.



## C H A P. X.

## Of Reduction.

**Reduction** is the Turning of a Quantity of one Denomination, into an equivalent Quantity of another Denomination. It is manifold.

The **Reduction**, which shall be here spoken of, is that of an *Integer* of one external Denomination into another. It is two-fold, *Descending* and *Ascending*.

**Descending Reduction**, is that, whereby an *Integer* of an *higher* or *greater* Denomination, is turn'd into an Equivalent of a *less* Denomination. And this is done by multiplying the Number to be reduc'd by so many *Units* of the less Denomination, as are Equivalent to one Unit of the greater Denomination. Thus, Pounds are turn'd into Shillings by multiplying by 20, Shillings into Pence by multiplying by 12, Pence into Farthings by multiplying by 4: Because, 4 Farthings = 1 Pence, and 12 Pence = 1 Shilling, and 20 Shillings = 1 Pound. So Yards are turn'd into Feet by multiplying by 3, and Feet into Inches by multiplying by 12:

Because,

Reduction, on, what.

2. Reduction of Integers into equivalent Integers, two-fold.

3. First, Descending Reduction, or the Reduction of Integers of a greater Denomination, into an Equivalent of a less Denomination.

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Because, 12 Inches = 1 Foot, and 3 Feet = 1 Yard.

4.  
Secondly, Ascending Reduction, or the Reduction of Integers of a lower Denomination, into an Equivalent of an Higher.

*Ascending Reduction*, is that, where an Integer of a lower Denomination, turn'd into an Equivalent of an Higher Denomination. And this is done by dividing the Integer to be reduc'd, by *any Units* of the lower Denomination, are equivalent to *one Unit* of the Higher. Thus, Farthings are turn'd into Pence dividing by 4, Pence into Shillings dividing by 12, and Shillings into Pounds by dividing by 20; for the Reason before assign'd in descending Reduction. Inches are turn'd into Feet by dividing by 12, Feet into Yards by dividing by 3; for the Reason likewise assign'd in descending Reduction.

5.  
The Use of the fore-going Reduction, illustrated.

These two Sorts of Reduction are Use in Multiplying or Dividing Integers of several external Denominations. If the Multiplier or Divisor consists of many Figures, then each Operation best perform'd by Reduction. For Instance: Suppose there were an 120 Persons, to each of which was to be paid 10*l.* 12*s.* 6*d.* 3*q.* And it is required to know, what Sum will pay them all; that is, what Sum 10*l.* 12*s.* 6*d.* 3*q.* multiplied by 120, amounts to. Now in order to multiply the Integers given Pounds, Shillings, Pence, and Farthings

must be all reduced into Qrs, etc.  
 lowest Denomination, i. e. Farthings.  
 which, according to the Method of do-  
 ing Reduction described, §. 3. is  
 done.

Which, according to the Method of  
 doing Reduction described, §. 3. is  
 done.

|               |             |                 |
|---------------|-------------|-----------------|
| Then          | 212         | Left, 2550      |
|               | 212         | 4               |
| 100 Shillings | 424         | 10200 Farthings |
| 12 given in.  | 212         | + 3 given.      |
| the Sum       | —           | —               |
| 12 Shillings  | 2544 Pence  | 10203 Farthings |
|               | + 6 given.  | —               |
|               | —           | —               |
|               | 2550 Pence. | —               |

It being thus found, that 10 l. 12 s.  
 4. 3 q. is equal to 10203 Farthings, I  
 multiply 10203 by 120, the Number of  
 persons propos'd.

|                    |
|--------------------|
| 10203 Farthings.   |
| 120                |
| —                  |
| 204060             |
| 10203              |
| —                  |
| 1224560 Farthings. |

Hence

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Hence, I learn, that the Sum requir'd to be known, is equal to 1224360 Farthings. It remains therefore to turn said Farthings into Pounds, Shillings and Pence, by ascending Reduc'tion. Which, according to the Method of the same describ'd, §. 4. is thus done.

4) 1224360 (306090 Pence.

024

24

036

36

00 Farthing.

Again, 12) 306090 (25507 Shillings

24

60

60

60

60

090

84

6 Pence.

Leff



Left, 220) 25507 (1275 Pounds,

20

55

40

150

140

107

100

7 shillings.

And thus it is found at Length, that  
 pay 10*l.* 12*s.* 6*d.* 3*q.*, to each of  
 Persons, I must have in all a Sum of  
 1275*l.* 7*s.* 6*d.* And whether no Error  
 committed in the fore-going Operati-  
 may be tried by working again  
 kwards, i. e. by dividing 1275*l.* 7*s.*  
 by 120, and seeing if the Quotient  
 be (as it ought) 10*l.* 12*s.* 6*d.*  
 In order whereunto, 1275*l.* 7*s.*  
 must be reduc'd all into Pence,

6.  
 The Proof  
 of the fore-  
 going Re-  
 duction.

1275

|       |        |       |
|-------|--------|-------|
| 1.    | Then   | 2.    |
| 2575  | 25507  |       |
| 20    | 12     |       |
| <hr/> | <hr/>  |       |
| 25500 | 31014  |       |
| +7    | 25507  |       |
| <hr/> | <hr/>  |       |
| 25507 | 506084 | Per   |
|       | +6     | gives |
|       | <hr/>  |       |
|       | 506096 | Per   |

Having thus found, that 12751  
6 d. = 306090 d., I divide 306090  
120, the Number of Persons assign'd.

$$\begin{array}{r} 2. \quad 2. \quad 2. \quad 2. \\ 120) 306090(2550 \quad 2550 \quad 2550 \quad 2550 \\ \underline{240} \phantom{00} \\ 660 \phantom{00} \\ \underline{600} \phantom{00} \\ 609 \phantom{00} \\ \underline{600} \phantom{00} \\ 90 \end{array}$$

Hence it appears, that the Sum  
Pence, which each of the 120 Persons  
to have, is 2550 Pence, and  $\frac{1}{4}$  of a Penny,  
ny, is  $\text{e. } 3$  Farthings over. Where  
I reduce the said Pence further into S  
Rings; Thus,

$$\begin{array}{r} 20 \overline{) 212} \end{array}$$

$$\begin{array}{r} 10 \end{array}$$

$$\begin{array}{r} 10 \\ 12 \end{array}$$

$$\begin{array}{r} 30 \\ 24 \end{array}$$

6 Pence.

Whence I find each of the 120 Persons to have 112 Shillings, and 6 Pence, Farthings over. I reduce, therefore, the said Shillings into Pounds;

$$\begin{array}{r} 20 \overline{) 212} \end{array}$$

And thus, I find, that each Person I have 10l. 12s. 6d. 3q, as was pos'd; and consequently, that the going Reduction is throughout right-perform'd. And I have insisted so upon the fore-going Example and Proof, because the Working thereof will

will serve to perfect the Learner in Multiplication and Division.

7:  
Another  
Example  
of the same  
Sort of  
Reduction.

In Reference to what was above (Q. 5. S. 14.) observ'd, the following Example shall be here adjoin'd, where is shewn, how much 245*l.* 6*s.* 10*d.* will make, being multiplied by 27, viz:

| <i>l.</i> | Then          | <i>s.</i>        |
|-----------|---------------|------------------|
| 245       |               | 4906             |
| * 20      |               | * 12             |
| 4900      | Shillings     | 9812             |
| + 6       | Shill. given. | 4906             |
| 4906      | Shillings.    | 58872 Pence.     |
|           |               | + 10 Pence given |
|           |               | 58882            |

It being thus found, that 245*l.* 10*d.* is equal to 58882 Pence, the equivalent Sum of Pence is to be multiplied by 27, viz:

$$\begin{array}{r}
 58882 \\
 \times 27 \\
 \hline
 412174 \\
 117764 \\
 \hline
 1589814
 \end{array}$$



It being found, that 58882 Pence  
multiplied by 27, make 1589814 Pence,  
the last Sum is to be reduc'd into Shil-  
lings and Pounds. Thus,

$$12 \overset{d.}{) 1589814} \overset{s.}{(132484}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 58 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

$$\begin{array}{r} 101 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ \hline \end{array}$$

$$\begin{array}{r} 54 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \text{ Pence.} \\ \hline \end{array}$$

H

Then

Then 20) 132484 (6624

120

---

124

120

---

48

40

---

84

80

---

4 Shillings.

---

And thus it is found, that 245 l. 6 s. 10 d., being multiplied by 27, amount to the Sum of 6624 l. 4 s. 6 d. Which is the very same Sum, that was found the other Way describ'd, §. 14, of Chap. 5, namely, by multiplying 245 l. 6 s. 10 d. (without *Reduction*;) first by 9, and then by 3, because  $9 \times 3 = 27$ . And therefore by comparing these two Ways together it appears how much shorter that other Way above describ'd is, than this *Reduction*. •

8.

Another  
Example,  
wherein  
both Multi-

It was likewise observ'd above, (§. 14, Chap. 5,) that Multipliers of several external Denominations, never occur but in Numbers of various Measures, viz. Yard

Yards, Feet, Inches, &c. And if you would in this Case work by *Reduction*, the Multiplier as well as Multiplicand are to be reduc'd into one Denomination. For Instance: Suppose 47 Feet, 8 Inches, are to be multiplied by 9 Feet, 4 Inches. First, 47 Feet, 8 Inches, are to be reduc'd into 572 Inches. Thus,

$$\begin{array}{r}
 47 \text{ Feet.} \\
 12 \text{ Inches.} \\
 \hline
 94 \\
 47 \\
 \hline
 564 \text{ Inches.} \\
 + 8 \text{ Inches given:} \\
 \hline
 572 \text{ Inches.} \\
 \hline
 \end{array}$$

Then 9 Feet and 4 Inches are to be reduc'd into 112 Inches, thus:  $12 \times 9 = 108$  Inches, and  $108 + 4 = 112$  Inches. Having thus reduc'd the Numbers given into one Denomination, viz. Inches, the respective Numbers of Inches are to be multiplied one into another, viz.

572 Inches.

112 Inches.

1144

572

57264064 Inches square.

Having thus found the Product (in square Inches) of 47 Feet, 8 Inches, multiplied into 9 Feet, 4 Inches; the said Product is to be reduc'd into square Feet by dividing it by 144, the Number of square Inches in one Foot square, viz.

144) 64064 (444 Feet square.

576

646

576

704

576128 Inches square.

And thus it is found, that 47 Feet, 8 Inches, multiply'd into 9 Feet, 4 Inches

will



will produce 444 Feet square, and 128 Inches square over. (\*) And thus much for the *Reduction* of *Integers* of one Denomination, into *Integers* of another Denomination.

The Sort of *Reduction*, which shall be mention'd here next, is that of *Fractions* of several Denominations, into equivalent Fractions of the same Denomination. Which is done by multiplying both Parts of each Fraction given, into the Denominator of the other Fraction. Thus,

9.  
Of Reduction of Fractions of several Denominations, into equivalent Fractions of the same Denomination.

(\*) According to what is above said, (§. 16. Chap. 3.) there is here taken Notice of by Way of Annotation, another Way of multiplying Numbers, when Multiplier, as well as Multiplicand is of several Denominations; which is call'd, *Cross Multiplication*, as on the side. Where the Product is the same, as is found the other Way by *Reduction*. Namely, here in *Cross Multiplication*, first 47 f.  $\times$  9 f. = 423 f. square. And likewise 8 Inches,  $\times$  4 Inches, = 32 Inches square. Then (47 f. =) 564 Inches,  $\times$  4 Inches, = 2256 Inches square; which divided by 144, (the Number of square Inches in a square Foot,) gives 15 f. square, + 96 Inches square. And likewise (9 f. =) 108 Inches,  $\times$  8 Inches, = 864 Inches square, = 6 Feet square. Which several particular Products and Sums put together make in all, 444 f. square, + 128 Inches square. For the Product of 47 Feet, 8 Inches,  $\times$  9 Feet, 4 Inches. This Sort of Working is call'd, *Cross Multiplication*, because 47 Feet are multiplied Cross-ways into 4 Inches, and 8 Inches Cross-ways into 9 Feet.

Feet. Inches.

47 . 08

09 . 04

---

423 . 00

00 . 32

15 . 96

06 . 00

---

444 . 128

H 3

$\frac{2}{3}$  and

$\frac{2}{3}$  and  $\frac{1}{4}$  are reduc'd to one Denominati-  
on, by multiplying  $\frac{2}{3}$  into 4, and  $\frac{1}{4}$  into  
3, viz.  $\frac{2}{3} \times 4 = \frac{8}{12}$ , and  $\frac{1}{4} \times 3 = \frac{3}{12}$ . For  
 $\frac{8}{12} = \frac{2}{3}$ , and  $\frac{3}{12} = \frac{1}{4}$ , that is, 2 bears the  
same Proportion to 3, as 8 to 12; and  
3 bears the same Proportion to 4, as  
to 12. And therefore it is the same  
add  $\frac{8}{12}$  to  $\frac{3}{12}$ , as to add  $\frac{2}{3}$  to  $\frac{1}{4}$ . To illu-  
strate the same by an Instance: It is ob-  
vious, that  $\frac{2}{3}$  of a Shilling, is 8 Pence  
and likewise  $\frac{1}{4}$  of a Shilling, is just the  
same. For to divide a Shilling into  
Parts, is to divide it into Groats; and  
Groats make 8 Pence. In like manner  
it is obvious, that  $\frac{1}{4}$  of a Shilling, is  
Pence; and  $\frac{3}{4}$  of a Shilling, is the same.  
For to divide a Shilling into 4 Parts,  
to divide it into Three-Pences, and three  
Three-Pences make 9d. Wherefore  
 $\frac{2}{3} + \frac{1}{4}$  of a Shilling,  $= \frac{8}{12} + \frac{3}{12}$  of a Shilling  
 $= \frac{11}{12}$ , i. e. 17 Pence, or 1 s. 5 d. And  
in *Subtraction*,  $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$ , i. e. of  
Penny. And so in *Division*,  $\frac{2}{3}$  divided  
by  $\frac{1}{4}$ , is the same as  $\frac{8}{12}$  divided by  $\frac{3}{12}$   
which is the same as 9d. divided by 8  
which is equal to  $1\frac{1}{8}$ , that is, 8d. is con-  
tain'd in 9d. Once, and there is a Penny  
over.

10.  
The Re-  
duction  
of a mixt

The third Sort of *Reduction*, to  
here taken Notice of, is the *Reduction*

a mi

*Mixt Fraction* into an *Improper*; which is done by multiplying the *Integer* into the Denominator of the Fraction, and adding the Product to the Numerator of the Fraction. Thus, the *Mixt Fraction*  $2\frac{1}{4}$  is reduc'd to the *improper Fraction*  $\frac{9}{4}$ .  $2 \times 4 = 8$ , and  $8 + \frac{1}{4} = \frac{9}{4}$ . So in *Al-*

$$\text{Algebra } C + \frac{SA}{R} = \frac{CR + SA}{R}.$$

On the other Hand, an *improper Fraction* is reduc'd to a *Mixt* or *Integer*, by dividing the Numerator by the Denominator. Thus,  $\frac{9}{4} = 2\frac{1}{4}$ . And  $\frac{8}{4} = 2$ . And it is to be observ'd, that, as this *Reduction* of an *improper Fraction* into a *Mixt*, or *Integer*, serves to shew the Value of the said *improper Fraction* more clearly; so the other *Reduction* of a *Mixt Fraction* into an *Improper* is necessary, as often as a *Mixt Fraction* occurs to be work'd with any other Fraction, for it must first be reduc'd to an *Improper*.

The *Reduction* of a *compound Fraction* into a *simple Fraction* is perform'd, by multiplying the Numerators one into the other, and also the Denominators. Thus,  $\frac{2}{3}$  of  $\frac{3}{4}$  of a Shilling, is equal to  $\frac{2}{4}$  of a Shilling, i. e. 3 Pence.

*Fraction, into an Improper.*

11.  
The Reduction of an improper Fraction, into a Mixt, or Integer.

12.  
Reduction of a compound Fraction, into a Simple.

13.  
The Reduction of a  
Fraction  
to lesser  
Terms.

A Fraction express'd by greater Numbers, is reduc'd to an equivalent Fraction express'd by less Numbers, by a Number that will divide both the Numerator and Denominator of the Fraction given. Thus,  $\frac{16}{4} = \frac{4}{1}$ , because  $16) \frac{16}{4}$  or thus,  $2) \frac{16}{4} | \frac{8}{2} | \frac{4}{1} | \frac{2}{1}$ . So  $\frac{3Aq}{6A}$  is

duc'd to  $\frac{A}{2}$ , by dividing both Terms

3A, This *Reduction* is of vast Use, being apparent, that Fractions may much more easily work'd in lesser than greater Terms. The Method of reducing a Fraction into the *least* Terms it can be reduc'd into, is here omitted, as being very frequently no less troublesome than to work by the Fractions given.

14.  
Reduction of Integers into Fractions of a given Denomination.

An *Integer* is reduc'd into an equivalent Fraction of a given Denomination by multiplying the *Integer* into the Denominator given, and taking the Product for the Numerator. Thus, the *Integer* is turn'd into a Fraction, whose Denominator is 4, namely,  $2 = \frac{4 \times 2}{4} = \frac{8}{4}$ . And this *Reduction* is necessary, as often as a Fraction is to be added to, or subtracted from an *Integer*. Thus,  $3 + \frac{3}{4} = \frac{12}{4} + \frac{3}{4}$

$= \frac{20}{4} = 5$ .



15.  
Reduction of com-  
mon Fra-  
ctions into  
Decimal.

Lastly, A common Fraction is turn'd into a Decimal, by dividing the Numerator, increas'd with Cyphers (as occasion requires) by the Denominator. Thus,  $\frac{1}{2}$  = .5. For 2) 10 (5, So  $\frac{1}{4}$  = .25; for 4) 100 (25. And  $\frac{1}{8}$  = .125; for 8) 1000 (125. So  $\frac{1}{3}$  = .333. For 3) 1000, &c. (333, &c. That is, the common Fraction  $\frac{1}{3}$  can't be reduc'd into a Decimal exactly equivalent thereto: However the Difference is (or may be by further dividing) render'd so small, as to be inconsiderable. Namely, there is not wanting  $\frac{1}{3000}$  to make the Decimal .333, exactly equivalent to the common Fraction  $\frac{1}{3}$ . And this is sufficient to our Purpose concerning Reduction.

CHAP.

## C H A P. XI.

Of Proportion, and more especially  
of the Rule of Three, or Golden  
Rule.

1.  
Proportion-  
al, what.

ANY one Number dividing any other Number, the Quotient thereby arising, shews the (\*) *Proportion* of the one to the other. Thus, because 2)6, therefore the greater Number 6 is said to be in a *triple* Proportion to the lesser 2, and 2 is said to be in a *Sub-triple* Proportion to 6.

2.  
Proportion-  
al Num-  
bers, what.

Four Numbers are said to be *proportional*, when the two first dividing the other, give the same Quotient, as the two last. Thus, because 2)6(3, and so 4)12(3, therefore 2 and 6, 4 and 12 are said to be proportional one to the other; which their Proportionality is wont to be thus express'd, viz.  $2 : 6 :: 4 : 12$  or  $6 : 2 :: 12 : 4$ .

3.  
A Remark  
concerning  
Proportion-  
als.

Of four Proportionals, some two always the Products or Quotients of the other two, multiplied or divided by so

---

(\*) This is by Mathematicians properly call'd the *Ratio*. But we use the word *Proportion* in the same Sense in common Speech.

and the same Number. Thus, in fore-going Example, 6 and 12 are Products of 2 and 4 multiplied by 3; and 4 are the Quotients of 6 and divided by 3.

ence, if four Numbers be proportionals, the Product of the two Extreams, of the 1st and 4th Terms or Numbers, is always equal to the Product of the Means, (i. e.) of the 2d and 3d Terms. Namely, in both Cases, the Ratios are in effect the same; and consequently, the Products must be the same. Instance;  $2 : (2 \times 3, \text{i. e.}) 6 :: 4 : (4 \times 3) 12$ . And therefore,  $2 \times 12 = 6 \times 4$ , because  $2 \times 4 \times 3 = 2 \times 3 \times 4$ .

On the fore-going Property of Proportionals, is founded the Rule of Proportion, which from its great Use in all Arts and Sciences, is commonly call'd the Golden Rule; as also the Rule of Three, from the three Proportionals given to find the Fourth. The Rule stands thus: the three Terms or Numbers given, let the Second multiply the Third, and the First divide the Product; the Quotient will be the fourth Proportional sought. For Instance: Let the three Numbers given be 2, 4, 6; and for the Fourth sought, Q. Wherefore,  $2 : 4 :: 6 : Q$ . Wherefore, by the fundamental Rule (given, 4.)  $2 \times Q = 4 \times 6$ , And therefore, forasmuch

4.  
The Foundation of the Golden Rule; and the Rule for proving Numbers to be Proportional.

5.  
The Golden Rule, or Rule of Three direct.

asmuch as  $\frac{2 \times Q}{2} = Q$ , it will follow

that  $\frac{4 \times 6}{2} = Q$ , that is, 2)24(12, the

Proportional sought. For 2 : 4 :: 6 :

6.  
Of placing  
the Terms  
in the  
Golden  
Rule.

If the Proportionals be of two several external Denominations, then they be so plac'd, as that the First and Third may be of one and the same Denomination, and likewise the Second and Fourth. For Instance : Supposing it be demanded, how much will be spent in 7 Days, at the Rate of spending five Shillings in seven Days ; the Terms must

d. s. d. s.

plac'd thus : 7 : 5 :: 365 : Q.

7.  
Further, of  
the same.

And if the Proportionals be expressed by more than two external Denominations, then they are to be reduc'd to Two. Thus, Supposing it be demanded, how much Money one shall spend in a Year at the Rate of five Shillings a Week, the four external Denominations here given (*viz.* Money, Shillings, Year, and Week) must be reduc'd to Two, *viz.* Shillings and Days, as in the Instance afore.

8.  
Direct  
Proportion,  
what.

If the Proportion runs so, that greater the third Term is, so much greater must be the Fourth ; or the less the third Term is, so much the less must be the Fourth ; then it is call'd direct Proportion ; and the fourth Proportion



is found out by the Rule afore laid  
 n. §. 5. Thus, because at what  
 Rate one spends, the more the  
 (which constitutes the third Term)  
 the more the Money spent (which  
 constitutes the fourth Term) will be;  
 e, in the fore-going Instance the Pro-  
 portion is Direct; and consequently, the  
 fourth Proportional is to be found by  
 Rule laid down, §. 5. Namely,

$$\begin{array}{l} d. \\ 365 : \left( \frac{365 \times 4}{7} \text{ i. e. } \right) 207 \frac{1}{7} :: 13 \frac{1}{2} : \frac{d.}{8 \frac{1}{2}} \text{ and a} \\ \text{over.} \end{array}$$

the Proportion runs so, that the  
 er the third Term is, so much the  
 must be the Fourth; or the *less* the  
 Term is, so much the *greater* must  
 the fourth Term; then it is call'd  
 (or *inverse*, or *reciprocal*) Proport  
 and the fourth Proportional is  
 out by a Rule somewhat different  
 the former, namely this: *The Pro-  
 of the first and second Term being di-  
 by the Third, the Quotient will be  
 fourth Proportional sought.* For In-  
 e: It has been found by the Rule  
 rect Proportion, that (round) 13 l.  
 60 Shillings will serve a whole Year  
 65 Days) at the Rate of five Shil-  
 a Week. I would know, how long  
 the

9.

Indirect  
 Proportion;  
 what; and  
 of the Rule  
 of Three  
 Indirect.

the same Sum will last at the Rate of four Shillings, or of four Shillings a Week. In which Case, the Terms must thus :

$$\begin{array}{ccccccc} s. & Days. & s. & & s. & Days. & i. \\ 5 & : 365 & :: 6 & : Q, & \text{or} & 5 & : 365 :: 4 \end{array}$$

Now, because it is obvious, that the greater the third Term is, so much the Fourth must be, *i. e.* the more Money is spent Weekly, the less Time the Sum 13 *l.* will hold out; or, as in the second Case, the less the third Term is, so much the greater the Fourth must be, *i. e.* the less Money is spent Weekly, the longer Time the Sum 13 *l.* will hold out: Hence it appears, that both the last Instances are to be work'd by Rule of indirect Proportion. Name

### C A S E I.

$$\begin{array}{ccccccc} s. & Days. & s. & & Days. \\ 5 & : 365 & :: 6 & : \left( \frac{365 \times 5}{6}, i. e. \right) & 304 \frac{1}{6} \end{array}$$

### C A S E II.

$$\begin{array}{ccccccc} s. & Days. & s. & & Days. \\ 5 & : 365 & :: 4 & : \left( \frac{365 \times 5}{4}, i. e. \right) & 456 \frac{1}{4} \end{array}$$

the Operation be duely perform'd according to the Rule of direct Proportion, when the Product of the Extreams as afore observ'd, §. 4.) will be equal to the Product of the Means. Thus, the Example, (§. 8.) of direct Proportion.

10.  
*The Proof of Working the Rule of direct Proportions*

$$365 \times 5 = 1825 = 260 \frac{1}{7} \times 7.$$

the Operation be duely perform'd according to the Rule of indirect Proportion, when the Product of the first and second Terms, will be equal to the Product of the third and fourth Terms. Thus, the Instances (contain'd, §. 9.) of indirect Proportion.

11.  
*The Proof of Working the Rule of indirect Proportion.*

$$365 \times 5 = 1825 = 304 \frac{1}{2} \times 6 :$$

$$\text{and } 365 \times 5 = 1825 = 456 \frac{1}{4} \times 4.$$

is worth Observation, that although the Rule of direct Proportion, be somewhat different from the Rule of indirect Proportion; and also the Proof of the former, be somewhat different from the Proof of the latter; yet, both the Rule of direct Proportion and its Proof, depend on the Rule and Proof of indirect Proportion. For indirect Proportion is reduc'd into Direct, by making the third Term of the former to be the first Term; in which the Operation and Proof of indirect

12.  
*The Rule of indirect Proportion, founded on the Rule of direct Proportion.*

indirect Proportion, will appear in  
to be the same with that of direct  
portion. Thus.

*s. Days. s. Days.*

Indirect Proport.  $5 : 365 :: 6 : 304$

*s. s. Days. Days.*

Direct Proport.  $6 : 5 :: 365 : 304$

Wherefore,

Proof of indir. Proport.  $365 \times 5 = 304$

Proof of direct Proport.  $365 \times 5 = 304$

13.  
*Of the  
compound  
Rule of  
Proportion.*

Hitherto we have spoken of the  
gle Rule of Proportion, or the Rule  
Three, namely, when no more than  
three Terms are given. It is now to  
observ'd, that if the first and third Terms  
have any Circumstances annex'd to them,  
and consequently there be more than  
three Terms given, then it is call'd  
compound Golden Rule, or the like;  
as much as such Questions consist at  
of two Proportionalities, and so may  
solv'd by working the simple Rule Twice.  
For Instance :

14.  
*An Exam-  
ple of the  
direct com-  
pound  
Rule.*

If four Men in three Months spend  
twenty Pounds, how much will six Men  
spend in twelve Months, at the same  
Rate? The Terms must stand thus

*Men. Months. l. Men. Months. l.*

$4 : 3 : 20 :: 6 : 12 : Q.$

W



Which may be resolv'd into these two  
single Operations, both Direct, viz,

$$\begin{array}{c} \text{Men.} \quad \text{l.} \quad \text{Men.} \quad \text{l.} \\ 4 : 20 :: 6 : \left( \frac{20 \times 6}{4} = \right) 30 \end{array}$$

$$\begin{array}{c} \text{Months.} \quad \text{l.} \quad \text{Months.} \quad \text{l.} \\ 3 : 30 :: 12 : \left( \frac{30 \times 12}{3} = \right) 120 \end{array}$$

But there is also another Way of  
working the same, namely, by reducing  
Terms given to Three, and so per-  
forming the Operation of the single Rule  
once. Thus,

$$\begin{array}{l} 3 : 20 :: 6 : 12 : Q, \text{ may be reduc'd} \\ 12 \text{ (i.e. } 4 \times 3) : 20 :: 72 \text{ (i.e. } 12 \times 6) : Q, \\ \text{then } 12 : 20 :: 72 : \left( \frac{72 \times 20}{12} = \right) 120. \end{array}$$

Whence it appears both Ways, that  
Q or Number sought is 120.

Proceed we now to the indirect com-  
pound Rule. Suppose then, 35 Shillings  
maintain three Men for six Days,  
how long will 180 Shillings, (i.e. 9 l.)  
maintain nine Men at the same Rate?  
Answer, ten Days. For first let us place  
Terms rightly thus, viz.

$$\begin{array}{c} \text{l.} \quad \text{Men. Days.} \quad \text{l.} \quad \text{Men. Days.} \\ 35 : 3 : 6 :: 180 : 9 : Q \end{array}$$

1 Then

Then by the single direct Rule, work thus :

$$36 : 6 :: 180 : \left( \frac{180 \times 6}{36} \text{, i.e.} \right) 30$$

And afterwards by the single indirect Rule, work thus :

| <i>Men.</i> | <i>Days.</i> | <i>Men.</i> | <i>Days.</i> |
|-------------|--------------|-------------|--------------|
| 3           | : 30         | ::          | 9 : 10,      |

Or the Terms given may be reduc'd Three, and the Number sought may be found by once Working according to the single Rule ; Thus,

$$540 (\text{i.e. } 180 \times 3) : 6 :: 324 (\text{i.e. } 36 \times 9) :$$

And this may suffice concerning the Golden Rule, (especially so call'd,) whether Single or Compound.

17.  
Of the  
Rule of  
Fellow-  
ship.

The *Rule of Society or Fellowship*, is indeed no other than the Golden Rule apply'd to Fellowship, or (as it is now a-days more frequently call'd) Partnership in Trading, or the like. For thereby is found each Partner's Share in the Gain or Loss, in Proportion to his Share in the common Stock. Namely, the Rule is this : *As the common Stock is to the common Gain or Loss, so is each Partner's Share in the common Stock, to his Share*

**common Gain or Loss.** For Instance:  
 If three Partners, one puts down 80*l*,  
 another 50*l*, and the third 40*l*. so, that  
 the common Stock is 170*l*. whereby,  
 suppose to be Gain'd or Lost 30*l*. It is  
 demanded, what must be each Man's  
 share in the Gain or Loss. Which is  
 find thus :

| Common<br>Stock. | Common<br>Gain or<br>Loss. | Shares in<br>Stock. | Shares in<br>Gain or<br>Loss. |
|------------------|----------------------------|---------------------|-------------------------------|
| 80               | 30                         | 80                  | 12.                           |
| 50               |                            | 50                  | 10.                           |
|                  |                            | 40                  | 08.                           |
|                  |                            | <u>170</u>          | <u>30</u>                     |
|                  |                            | Proof 170           | 30                            |

The Proof of the Operation consists in  
 the particular Shares in the Stock, ma-  
 king up the whole Stock ; and the parti-  
 cular Shares in the Gain or Loss, making  
 the whole Gain or Loss.

If, besides the particular Shares them-  
 selves, Consideration be had of the Times  
 putting down the Shares, as is requi-  
 red when the Shares are not put down  
 together ; then it is call'd the *compound*  
*Rule of Fellowship*, and is perform'd as  
 the compound Rule of direct Proportion.  
 Namely, supposing there be three Part-  
 ners, one of which puts down 10*l*. for  
 1 2 four

18.  
 of the  
 compound  
 Rule of  
 Fellowship.

four Months, another 12 l. for 2 Months, and the third 20 l. for one Month; and thereby be Gain'd 40 Shillings, or two Pounds. What must be each Man's Share in the said Gain?

| Sum of common Stock, multiply'd into common Gain. | Sum of Gain. | Particular Shares<br>× partic. Times. | Particular Gains. |                      |
|---------------------------------------------------|--------------|---------------------------------------|-------------------|----------------------|
|                                                   |              | 40 :                                  | 80                |                      |
|                                                   |              |                                       | 84                |                      |
| 84 :                                              | 2 :          | 24 :                                  | 48                |                      |
|                                                   |              |                                       | 84                |                      |
|                                                   |              | 20 :                                  | 40                |                      |
|                                                   |              |                                       | 84                |                      |
|                                                   |              |                                       |                   | $\frac{168}{84} = 2$ |
| Proof                                             |              | <u>84</u>                             | <u>2</u>          |                      |

19.

The Rule of Alligation, and of False, why here omitted.

As for the Rule of *Alligation*, it being not of frequent Use, it is here pass'd by. And because what may be done by the Rule of *False*, may more easily and universally be done by the Rule of *Equation* in *Algebra*, it likewise is here omitted, but set down under the Rule of *Equation* by Way of Annotation.

20.

Direct, Altern, Inverse, Compound, Divisive,

Proceed we therefore to observe, (which is of much greater Use in *Mathematics* viz.) that if four Quantities be direct Proportional, they will be so likewise Alternately, and Inversely, and Compoundly.



ity, and *Divisively*, and *Conversely*, and  
*For Instance*:

Converse,  
 and mixt  
 Proportions,  
 what.

|                          |   |                                |  |
|--------------------------|---|--------------------------------|--|
| Direct<br>Proport.       | { | A : a :: B : b                 |  |
|                          | { | 8 : 4 :: 6 : 3                 |  |
| Altern.<br>Proport.      | { | A : B :: a : b                 |  |
|                          | { | 8 : 6 :: 4 : 3                 |  |
| Inverse<br>Proport.      | { | a : A :: b : B                 |  |
|                          | { | 4 : 8 :: 3 : 6                 |  |
| Compound-<br>ed Proport. | { | A + a : a :: B + b : b         |  |
|                          |   | 8 + 4 : 4 :: 6 + 3 : 3         |  |
|                          | { | Or,                            |  |
|                          |   | A + B : B :: a + b : b         |  |
|                          |   | 8 + 6 : 6 :: 4 + 3 : 3         |  |
| Divided<br>Proport.      | { | A - a : a :: B - b : b         |  |
|                          |   | 8 - 4 : 4 :: 6 - 3 : 3         |  |
|                          | { | Or,                            |  |
|                          |   | A - B : B :: a - b : b         |  |
|                          |   | 8 - 6 : 6 :: 4 - 3 : 3         |  |
| Converse<br>Proport.     | { | A : A + a :: B : B + b         |  |
|                          |   | 8 : 8 + 4 :: 6 : 6 + 3         |  |
|                          | { | Or,                            |  |
|                          |   | A : A + B :: a : a + b         |  |
|                          |   | 8 : 8 + 6 :: 4 : 4 + 3         |  |
| Ext Pro-<br>portion.     | { | A + a : A - a :: B + b : B - b |  |
|                          |   | 8 + 4 : 8 - 4 :: 6 + 3 : 6 - 3 |  |
|                          | { | Or,                            |  |
|                          |   | A + B : A - B :: a + b : a - b |  |
|                          |   | 8 + 6 : 8 - 6 :: 4 + 3 : 4 - 3 |  |

For in each  
 Case, the  
 Product of  
 the Ex-  
 tremes, is  
 equal to  
 the Pro-  
 duct of the  
 Means.

21. *Proportion, two-fold.*

It is next to be observ'd, that Proportion is distinguish'd into *Arithmetical* and *Geometrical*.

22. *Arithmetical Proportion, what.*

*Arithmetical* Proportion is, when Terms do exceed one the other, by having the same Number *added* to (or *subtracted* from) them. And it is so term'd because Numbers call'd in *Greek* *Arithmoi* do exceed one the other in their natural Series or Order, according to this Proportion, viz. 1,  $(1+1=) 2$ ,  $(2+1=) 3$ ,  $(3+1=) 4$ ,  $(4+1=) 5$ , &c.

23. *Geometrical Proportion, what.*

*Geometrical* Proportion is, when Terms do exceed one the other, by being *multiplied* (or *divided*) by the same Number. Thus, 2,  $(2 \times 2 =) 4$ ,  $(4 \times 2 =) 8$ ,  $(8 \times 2 =) 16$ ,  $(16 \times 2 =) 32$ , (&c.) are to be in *Geometrical* Proportion, namely, because *Geometrical* Numbers, *Squares*, *Cubes*, &c. (of which in the following Chapter) are in this Proportion one to the other.

24. *The Antecedent and Consequent in a Proportion, what.*

It is also to be known, that in a two Terms, whether of *Arithmetical* or *Geometrical* Proportion, that which is *transferr'd* to the other, and is wont to be plac'd First, is thence call'd the *Antecedent*, the other the *Consequent*. Hence all Numbers, whose Antecedents stand alike to their respective Consequents, are Proportional one to the other. Whether

whether in *Arithmetical* Proportion, as

Conf. Ant. Conf. Ant. Conf.  
 $4 :: 12 : 9 :: 20 : 17$ , &c. ; or  
 Ant. Conf.

*Geometrical* Proportion, as  $2 : 6 ::$

Conf. Ant. Conf.  
 $9 :: 4 : 12$ , &c.

And if in either Proportion, that

Term, which was afore the *Consequent*,

*Continuè*, or presently made the *Ante-*

*cedent*, then the said Terms are said to

be in *Continual* Proportion ; as in *Arith-*

*metical* Proportion,  $4 : 7 :: 7 : 10$  ; and

*Geometrical* Proportion,  $4 : 8 :: 8 : 16$ .

Again, if the said *Continual* Proporti-

on, whether *Arithmetical* or *Geometrical*,

be actually (*progreditur*, i. e.) pro-

ceeds, or is conceiv'd to proceed beyond

our Terms, then it is peculiarly call'd

*Progression*. Thus,  $2 : 4 : 6 : 8 : 10 : 12$ ,

&c. constitute an *Arithmetical* *Progressi-*

*on*, and  $2 : 4 : 8 : 16 : 32 : 64$ , &c.

constitute a *Geometrical* *Progression*.

In *Arithmetical* *Progression*, any Term

requir'd may be made (without making

the *Antecedent* Terms) by (+) adding

25.

*Continual*  
*Proportion,*  
*what.*

26.

*Progressi-*  
*on, what.*

27.

To find  
 any Term  
 in *Arith-*  
*metical*  
*Progression.*

(+) The Rule may be denoted in short by Symbols,  
 viz.  $T = DX + a$ , where T denotes the Term sought,  
 in Distance from the first Term, X the Difference of  
 Terms, DX the Distance multiplied into the Dis-  
 tance, a the first Term.

I 4

the

the first Term to the Product of the Difference, (*i. e.* the Number whereby the Terms exceed one another,) multiplied into the Distance of the Term requir'd from the First. For Instance: In this *Arithmetical* Progression, 2, 4, 6, &c. where the first Term is 2, and the Difference of each Term is 2, we would know what will be the twentieth Term; whose Distance consequently from the first Term is 19. Wherefore  $19 \times 2 = 38$ , and  $38 + 2 = 40$ . Which therefore will constitute the twentieth Term requir'd in the Progression propos'd.

28.  
To find  
the Sum of  
any *Arith-*  
*metical*  
*Progression.*

The Sum of any *Arithmetical* Progression, is found by (||) multiplying the Sum of the first and last Terms, into the Number of all the Terms, and dividing the Product by 2. For Example; Suppose 20 Stones were each plac'd at a Statue Pole's Distance one from the other, and the first likewise a Pole's Distance from the Place, whither all the Stones were to be brought singly in Order one after the other. It is demanded, how much he would go in all, that should

---

(||) The Rule may be express'd in short by Symbols, *viz.*  $Z = a + l \times T$ ; where Z denotes the Sum, *a* the first Term, *l* the Last,  $a + l$  the Sum of the first and last Terms, T the Number of Terms.



bring the Stones to the said Place ?  
 Answer, 420 Poles Length, which make  
 the statute Mile, and 100 Poles (i. e.  
 5 Furlongs and an Half) over. For in  
 going from and coming back to the Place  
 in order to bring the first Stone, he will  
 go two Poles Length, and in going and  
 coming in order to bring the second  
 Stone, he will go four Poles Length,  
 and likewise in order to bring the third  
 Stone, he will go six Poles Length, &c.  
 Hence it is evident, that 2 will be the  
 Term of this Progression, and also  
 the Difference of all the Terms from each  
 other. And consequently it will be  
 found by the Rule in the fore-going  
 Section, that in going and coming to  
 bring the twentieth or last Stone, he will  
 go 40 Poles Length. Which being  
 known, the Sum of all he has gone in  
 bringing all the said Stones to the said  
 Place, may be found by the Rule de-  
 scrib'd here in this Section. Namely,  
 $2 + 40 = 42$ , and  $42 \times 20 = 840$ , and  
 $840 \div 2 = 420$ .

Any Term of Geometrical Progression  
 may be found thus. Having made so many as  
 you please of the first Terms of your  
 Progression, by continually multiplying  
 the fore-going Term into the given Ra-  
 tio (i. e. the common Number, by  
 which

29.  
 To find  
 any Term  
 in Geome-  
 trical Pro-  
 gression.

which each Term of the Progression multiplied to produce the next following Term,) number the said Terms according to their respective Order, the Figure whereby the Terms are thus numbered being call'd their *Indices*. Then add together any two *Indices*, whose Sum less'n'd by One, will equal the Index of the Term sought. The two Terms which stand under the two *Indices*, added together, being likewise multiplied one into the other, and their Product divided by the first Term, the Quotient will be the Term sought. For Instance Suppose there is Occasion for a *Geometrical* Progression, whose first Term shall be One, and its *Ratio* 2, and where the 24th Term is principally to be known. Hereupon I make some of the first Terms, and write them and their *Indices*, as follows ;

|                 |    |    |    |    |     |     |     |      |     |
|-----------------|----|----|----|----|-----|-----|-----|------|-----|
| <i>Indices,</i> | 1  | 2  | 3  | 4  | 5   | 6   | 7   | 8,   | &c. |
| <i>Terms,</i>   | 1. | 2. | 4. | 8. | 16. | 32. | 64. | 128, | &c. |

Now, because no two of these *Indices* put together, and less'n'd by One, will make 24, the Number of the Term sought, therefore I first find the 12th Term : Namely, because the  $6+7-1=12$ , therefore I multiply

32, and the Product is 4096; which (because the first Term, whereby it is to be divided, is one) is also the 12th Term sought.

$$\begin{array}{r} 32 \\ 128 \\ \hline 192 \\ 2048 \end{array}$$

Having found the 12th Term, I proceed next to find the 13th Term, (because these two together will give me the 24th Term sought). Now the  $6+8-1=13$ , therefore multiply 128 by 32, and the Product is 4096; which for Reason afore assign'd is also the 13th Term.

$$\begin{array}{r} 128 \\ 32 \\ \hline 256 \\ 384 \\ \hline 4096 \end{array}$$

30.  
Further of  
the same.

Having thus found the 12th and 13th Term, because the Indices  $12+13-1=24$ , the Number or Index of the Term requir'd, therefore I multiply 4096 by 2048, and the Product 8388608 (for the Reason afore assign'd, without more ado) is the 24th Term, or the Term sought.

$$\begin{array}{r} 4096 \\ 2048 \\ \hline 32768 \\ 16384 \\ 81920 \\ \hline 8388608 \end{array}$$

31.  
Further  
yet of the  
same.

The Sum of any Geometrical Progression is found thus: (\*) Multiply the last

32.  
To find  
the Sum

(\*) The Rule may be express'd in short by Symbols, viz.  $Z = \frac{\beta\omega - a^2}{\beta - a}$ . Where Z denotes the Sum, a

first Term,  $\beta$  the second Term,  $\omega$  the last Term,  $a^2$  Square of the first Term,  $\beta\omega$  the second Term multiplied into the last.

Term

of any  
Geometri-  
cal Pro-  
gression.

Term by the Second, and from the Product, substract the Square of the Term. Then by the second Term, send by the *Substraction* of the first Term divide the Remainder, and the Quotient will be the Sum of the Progression. Instance: 2. 4. 8. 16. 32. are so many Terms of a Geometrical Progression which added together, make 62. And so much they will be found to make Working according to the Rule here down, viz.  $32 \times 4 = 128$ , and  $128 - 124 = 4$ , and then 124 divided by (4 - i. e.) 2 will give 62.

33.

An Exam-  
ple of the  
said Rule.

Having thus shewn, that the afore-  
Rule holds true, I shall now shew, by the same Rule is solv'd that common Question, viz. what the Price of a Horse will amount to, suppose he sold at the Rate of a Farthing for the first Nail, two Farthings for the second Nail, four Farthings for the third Nail, and so on doubling the Price of each Nail, for as many Nails as there be in his four Shooes, supposing the Nails be in all 24, viz. six in each Shoe. Now, according to the State of the Question, it is evident, that the First Term of the Geometrical Progression relating thereunto is One, and the Ratio of the several Terms in the said Progression is 2, as in §. 29. whence it followeth



according to the Rule, §, 29, for  
 any Term in Geometrical Progression  
 the last or 24th Term of the Pro-  
 portion relating to this Question, must  
 (as is found, (§. 31.) 83886c8.  
 Therefore, according to the Rule, (§.  
 for finding the Sum of any Geome-  
 trical Progression; the Sum here requir'd  
 be this, viz. 16777215 Farthings,  
 which by Reduction will be found equal  
 7476l. 5s. 3d. 3q. much too great  
 a Price for the best of Horses.

Now, supposing only a Pin to be paid  
 for the first Nail, and consequently for  
 the 24th Nail to be paid 16777215 Pins,  
 the said Pins to be worth no more  
 than a Groat a Thousand; yet it will  
 appear by Reduction, that, according to  
 the Rate, the Price of an Horse so sold,  
 amounts to 279l. 12s. 4d. and up-  
 wards; a sufficient Price for the best of  
 Horses. And thus much for such Rules  
 of Arithmetick, as relate to Proportion.

34.

Further of  
the same.

C H A P.

## C H A P. XII.

Of the Extraction of the Square  
cube Root.

1.  
A Root,  
and a  
Power,  
what.

**B**y a Root is signified any Number or Algebraical Quantity, from which more or less multiplied into it self arise Products, which are distinguished (from other Products arising from Multiplication of two different Numbers or Quantities one into the other) by Name of Powers. Thus, 3 is said to be the Root of 9, and 27, &c. because  $3 \times 3 = 9$ , and  $3 \times 3 \times 3 = 27$ . So A is said to be the Root of AA or  $A^2$ , and AAA or  $A^3$ , &c. because  $A \times A = AA$ ,  $A \times A \times A = AAA$ .

2.  
The Dimensions  
of Powers,  
how reckon'd.

As often as any Power involves a Root, so many Dimensions the said Power is said to be of. Thus, 9 (or  $3 \times 3$ ) is said to be a Power of two Dimensions 27 (i. e.  $3 \times 3 \times 3$ ) of three Dimensions &c. So AA or  $A^2$  has Two, AAA or  $A^3$  has three Dimensions: Where it is also to be observ'd by the Way, that  $A^1$ ,  $A^2$ , &c. the vertical Figures from their Use, call'd the *Indices* of Dimensions.

Each Power is distinguish'd by a peculiar Name, according to the Number of Dimensions. Thus,  $A^1$  or 9, is call'd a secundan Power, or a Power of second Order,  $A^1$  or 27, a tertian Power, or Power of the third Order,

3.  
The peculiar Names of Powers, whence taken.

The several Powers do also borrow other Names from Geometrical Quantities. As, a Root is otherwise call'd by a geometrical Name, a Side; a secundan Power is otherwise call'd by a Geometrical Name, a Square; a tertian Power, a Cube, (\*) &c. Hence,  $Aq$  is the same as  $A^2$  or  $A^1$ ,  $Ac$  the same as  $AAA$  or  $A^3$ . And 9 is said to be the Square, 27 the Cube of 3, which is call'd the Side of 9 and 27, &c.

4.  
Other Geometrical Names of the Powers.

It was observ'd, Chap. 9. §. 13, that two Examples of Algebraical Mathematics therein contain'd are of great Use, when thoroughly understood; for as thereby the Extraction of the Square and cube Root is render'd more easy to be apprehended, and consequently to be perform'd. Namely, the Proposition of  $A+E$  multiplied into  $A+E$ , (Example 7,) viz.  $Aq+2AE+Eq$ , shews several Members or Parts, whereof

5.  
A thorough Understanding of Algebraical Multiplication, is of great Use towards the Extraction of the Square Root.

) See the following Names and Characters in Notes to Chap. 9. of my Latin Arithmetick.

consists

consists the Square of any binomial Root, *i. e.* of any Root consisting of two Figures or Quantities. For  $Aq + 2AE$  express'd in Words, denotes thus *viz.* that the Square of every binomial Root is made up of ( $Aq$ , *i. e.*) the Square of ( $A$ , *i. e.*) the greater Figure or Quantity, and of ( $Eq$ , *i. e.*) the Square of ( $E$ , *i. e.*) the lesser Figure or Quantity, and also of ( $2AE$ , *i. e.*) the Double of the Product of both Figures or Quantities, multiplied one into the other.

6.

And also  
to the Ex-  
traction  
of the cube  
Root.

In like manner, the Product  $Aq + 2AE + Eq$ , multiplied again by the Root  $A + E$ , (*Example 8*,)  $Ac + 3AqE + 3AEq + Ec$ , shews the several Members, whereof consists the Cube of any binomial Root.  $Ac + 3AqE + 3AEq + Ec$ , denotes in Words thus much, *viz.* that the Cube of every binomial Root is made up of ( $Ac$ , *i. e.*) the Cube of ( $A$ ), the greater Figure or Quantity; and of ( $Ec$ , *i. e.*) the Cube of ( $E$ ), the lesser Figure or Quantity; also of ( $3AqE$ , *i. e.*) the Triple of the Product of the Square of the greater Figure or Quantity, multiplied into the Lesser; and likewise of ( $3AEq$ , *i. e.*) the Triple of the Product of the Square of the Lesser Figure or Quantity, multiplied into the Greater.



From what has been said, appears  
 sufficiently one Excellency of *Algebraical*  
 Operations; inasmuch as any Operation  
 commonly perform'd, serves as a Rule for  
 performing all other Operations of the  
 Kind; and that too such a Rule, as  
 Understanding may much sooner ap-  
 prehend, it being express'd by a few  
 Symbols or Characters lying all in one  
 Row, than when the same Rule is run  
 into a Multitude of Words; necessa-  
 rily to express it, but yet Con-  
 founding, rather than informing the Un-  
 derstanding. For which Reason I choose  
 to teach the Manner of Extracting the  
 Square and cube Root, by the said *Alge-*  
*braical* Symbols, rather than by Words,  
 in the following Part of this Chapter.

Whereas, then it has been shewn, how  
 the binomial Root, denoted by  $A+E$ ,  
 being multiplied into it self produces a  
 Square, consisting of three several Mem-  
 bers, denoted by  $Aq+2AE+Eq$ ; it  
 becomes not difficult, having the  
 form of any binomial Root, denoted  
 by  $Aq+2AE+Eq$ , to resolve it into its  
 Root, or as it is commonly express'd, to  
 extract its Root, denoted by  $A+E$ .  
 Namely, such a Square is resolv'd, first  
 taking out of it  $Aq$ , and taking  $A$  for  
 the greater Quantity of the Root sought;  
 then dividing the next Member  $2AE$

7.  
 One Excel-  
 lency of  
 Algebra.

8.  
 The Rule  
 for Ex-  
 tracting  
 the Square  
 Root;  
 when it  
 consists but  
 of two Fi-  
 gures or  
 Quanti-  
 ties.

of the Square, by  $2A$  the Double of greater Quantity, and taking the mainder  $E$  for the less Quantity of Root sought. For  $E$  being multiplied by  $2A$ , makes  $2AE$ ; and  $E \times E$ , makes  $E^2$ . And  $2AE + E^2$  being subtracted from the Residue of the Square, as that  $A^2$  was taken away, there remains Nothing. Wherefore,  $A + E$  is the Root sought. For Instance :

$$\begin{array}{r}
 \text{Subtract} \quad A^2 + 2AE + E^2 \quad (A + E)^2 \\
 \underline{A^2} \\
 \text{Divide by } 2A) \quad 0 + 2AE + E^2 \\
 \underline{2AE + E^2} \\
 0
 \end{array}$$

9.  
The same  
illustrated  
by an Ex-  
ample in  
Numbers.

So in Numbers. For, suppose Square 144 to be propos'd, in order to find its Root; I consider, (according to what has been observ'd.) that it consists of three Members, answering to the three, viz.  $A^2$ ,  $2AE$ , and  $E^2$ , of which  $E^2$  (according to the Order of the Members, and what is illustrated by Example VII. Chap. 9. §. 13.) refers always to the right-hand Figure, and the other two  $2AE$  and  $A^2$ , to the two next following Figures of the Square in their respective Order. Whence it is usual to distinguish any Square into its respective Members.

## EXAMPLE I.

$A_q$ , (i.e. the greatest Sq. in  $\tau$ )  $\dots$   $i$

2A) 2AE (E . . 2)      044  
 $\left. \begin{array}{r} 2AE = 4 \\ E_7 = 4 \end{array} \right\} \text{Subtract } \therefore \underline{44}$   
44  
 0

### EXAMPLE II.

$q$ , (i.e. the biggest  $S_q$  in  $32$ ) ... 25

$$\begin{array}{r} 2A) 2AE \text{ (E...to)} \quad 749 \\ 2AE \quad 70 \\ E_1 \quad 49 \\ \hline 749 \end{array} \left. \vphantom{\begin{array}{r} 2A) 2AE \text{ (E...to)} \\ 2AE \\ E_1 \end{array}} \right\} \text{Subtract } \begin{array}{r} 749 \\ \hline 0 \end{array}$$

It is to be observ'd, that, if the Square pos'd consists of more than four Figures, then the Root to be extracted consist of more than two Figures; the fore-going Operation is to be re-  
ted, as in the following Example.

Ka

**E X-**

## EXAMPLE III.

Squ. propos'd . . . 327184 (572 Root)

Subtract  $Aq$  . . . . . 29

$2A$   $2AE$  (E . . . . 10) 771

$2AE$  . . 70  
 $Eq$  . . . 49  
 ————  
 749

Subtract . . 749

---

$2A$   $2AE$  (E . . . 144) 2284

$2AE$  . . 228  
 $Eq$  . . . . 4  
 ————  
 2284

Subtract . . 2284

---

0

The only Particular here observ'd is, that having found the two Figures of the Root sought, by the first Operation; in the second Operation, both said Figures are to be esteem'd as A, the third Figure 2 of the Root is to esteem'd as E. And if the said Root consist of four Figures, and so the Operation were again to be repeated, all the three Figures here already found are to be esteem'd as A, and the Fourth to be found as E. And so on.

10.  
Of Ex-  
tracting  
the cube  
Root.

As before has been shewn, the Method of Extracting the square Root of a common Number, by the Algebraical Square  $Aq + 2AE + Eq$ ; so we proceed now to shew, the Method of Extracting the



of any common Number, by the  
 p of the *algebraical* Cube  $A^3 + 3AEq + E^3$  Namely, this Cube  
 resolv'd, first by taking out of it  $Ac$ ,  
 taking  $A$  (the Side of  $Ac$ ) for the  
 later Quantity of the Root sought;  
 then dividing the next Member  
 $3AEq$  by  $3Aq$ , and taking the Remainder  
 for the less Quantity of the Root  
 sought. For  $E$  being multiplied by the  
 divisor  $3Aq$ , makes  $3AEq$ , and  $E^3 = 3AEq$ , and  $E \times E \times E = E^3$ . But now  
 $3AEq + 3AEq + E^3$  being subtracted from  
 what remains of the Cube after that  $Ac$   
 subtracted, there remains Nothing.  
 Therefore,  $A + E$  is the cubical Root  
 sought. For Instance:

$$\begin{array}{r}
 A^3 + 3AEq + 3AEq + E^3 \quad (A + E \\
 \text{subtract} \quad A^3 \\
 \hline
 0 + 3AEq - 3AEq - E^3. \\
 3AEq - 3AEq - E^3
 \end{array}$$

in Numbers. For, suppose the  
 of the Cube 1728 to be demanded.  
 Consider it consists of four Members,  
 answering to these four, viz.  $Ac$ ,  $3AEq$ ,  
 $3AEq$ , and  $E^3$ ; and that so, as  $E^3$  be-  
 comes to the right-hand Figure 8, and so  
 subtract  $3AEq$ ,  $3AEq$ , and  $Ac$  to the o-  
 ther

II.  
*The same  
 illustrated  
 by an Ex-  
 ample in  
 Numbers.*

ther three Figures in Order. When  
as it is usual, I distinguish the given  
into the said respective Members,  
pointing the right-hand Figure first,  
then every third Figure, as 1728,

so proceed to the Extraction of  
Root, as follows :

**EXAMPLE IV.**

$$\begin{array}{r}
 1728 \text{ (12)} \\
 \text{Subst. Ac, (i. e. biggest Cube in 1)} \dots 1 \\
 \hline
 3A9) 3A9E \text{ (E} \dots 3) \quad 0728 \\
 \underline{3A9E = 6} \\
 3AE9 = 12 \\
 \underline{E = 8} \\
 \hline
 728 \\
 \hline
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Subtract} \dots 728 \\ \hline 0 \end{array}$$

**EXAMPLE V.**

$$\begin{array}{r}
 185193 \text{ (57)} \\
 \text{Subst. Ac, (i. e. biggest Cube in 185)} \dots 185 \\
 \hline
 9A9) 9A9E \text{ (E} \dots 75) \quad 60193 \\
 \underline{9A9E = 529} \\
 9AE9 \dots 739 \\
 \underline{E \dots 943} \\
 \hline
 60193 \\
 \hline
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Subtract} \quad 60193 \\ \hline 0 \end{array}$$

**EXAMPLE VI.**

187149248 (972

Su. Ac, (i. e. biggest Cube in 187) . . . 125

$$\begin{array}{r}
 3Aq) \quad 3AqE \quad (E \quad . . . . . 75) \quad 62149 \\
 \underline{3AqE} \quad 325 \\
 3AEq \quad 795 \\
 \underline{Ec} \quad 343 \\
 60193
 \end{array}
 \left. \vphantom{\begin{array}{r} 3Aq) \quad 3AqE \quad (E \quad . . . . . 75) \quad 62149 \\ \underline{3AqE} \quad 325 \\ 3AEq \quad 795 \\ \underline{Ec} \quad 343 \\ 60193 \end{array}} \right\} \text{Subtract} \quad 60193$$

$$\begin{array}{r}
 3Aq) \quad 3AqE \quad (E \quad . . . . . 9747) \quad 1956248 \\
 \underline{3AqE} \quad 19494 \\
 3AEq \quad 684 \\
 \underline{Ec} \quad 8 \\
 1956248
 \end{array}
 \left. \vphantom{\begin{array}{r} 3Aq) \quad 3AqE \quad (E \quad . . . . . 9747) \quad 1956248 \\ \underline{3AqE} \quad 19494 \\ 3AEq \quad 684 \\ \underline{Ec} \quad 8 \\ 1956248 \end{array}} \right\} \text{Subtract} \quad 1956248$$

The only Particular here to be ob-  
 v'd, is the same with what was ob-  
 v'd in Reference to the square Root,  
 Example 3; namely, that having by the  
 Operation found the two Figures 57  
 the Root sought, both the said Fi-  
 gures in the second Operation are to be  
 esteem'd as A; and the third Figure 3  
 the Root is to be esteem'd as E. And  
 the said Root did consist of four Fi-  
 gures, and so the Operation were again  
 repeated, then all the three Figures  
 already found are to be esteem'd as A,  
 the Fourth remaining to be found  
 as E. And so on, according to the

Number of the Figures, which the Root consists of.

12.  
A surd  
Root,  
what.

If, after that the Square or cube Root has been Extracted out of any Number propos'd, according to the foregoing Rules, there remains any Part of the Number; then the said Number is not an exact Square or Cube, but so much over as is denoted by what remains of the said Number. And it often happens that in mathematical Operations, there is Occasion to look on such Numbers and Quantities, as Squares or Cubes, which yet are not so; and consequently, whose Roots can't be express'd in Numbers (whence they are call'd *surd* Roots,) and are wont to be denoted thus, viz.  $\sqrt[3]{6}$

or  $\sqrt{6}$ , or simply  $\sqrt{6}$ , denotes the Root of the Number 6 taken as a Square.

In like manner,  $\sqrt[3]{6}$ , or  $\sqrt[3]{6}$ , denotes the Root of the Number 6 taken as a Cube. And therefore, the Square of the  $\sqrt{6}$ , is 6; the Cube of the *surd*  $\sqrt[3]{6}$  likewise 6; and so of any other *Surd*.

13.  
How to  
extract  
the Root  
of a qua-  
dratick or  
cubical  
Fraction.

As a Fraction is squar'd or cub'd, by Squaring or Cubing its Numerator and Denominator; so a quadratick or cubical Fraction is resolv'd into its Root, by solving its Numerator and Denominator into their respective Roots. Thus, Squ



are of  $\frac{1}{2}$  is 1, the Cube of  $\frac{1}{2}$  is  $\frac{1}{8}$   
 and on the other Hand, the Root of the  
 are  $\frac{1}{8}$ , or of the Cube,  $\frac{1}{2}$  is  $\frac{1}{2}$ . If the  
 Root of the Fraction can't be extracted,  
 express'd in Numbers, then 'tis call'd a  
 surd Fraction: as the Root of  $\frac{1}{2}$  taking  
 for a Square, is  $\sqrt{\frac{1}{2}}$ , the Root of  $\frac{1}{8}$  tak-  
 ing it for a Cube, is  $\sqrt[3]{\frac{1}{8}}$ .  
 The Squares and Cubes of the nine  
 Digits or single Figures ought to be so  
 well known, as that at the bare Sight of  
 them their respective Roots should be  
 perfectly known. To which End the  
 said Digits are here set down, with their  
 respective Squares and Cubes on their  
 right.

Now from  
 this Table  
 it may be  
 seen, that  
 the Roots  
 of the  
 Squares  
 and Cubes  
 of the nine  
 Digits.

14.  
 Of the  
 Squares  
 and Cubes  
 of the nine  
 Digits.

| Root. | Square. | Cube. |
|-------|---------|-------|
| 1     | 1       | 1     |
| 2     | 4       | 8     |
| 3     | 9       | 27    |
| 4     | 16      | 64    |
| 5     | 25      | 125   |
| 6     | 36      | 216   |
| 7     | 49      | 343   |
| 8     | 64      | 512   |
| 9     | 81      | 729   |

These Squares and Cubes of the nine  
 Digits, make up one of those numbering  
 Tables, which commonly go by the Name  
 of Naper's Bones, as being of good Use,  
 together

15.  
 Extraction  
 of the  
 Square or  
 Cube Root,  
 is render'd

more easy  
and quick  
by Naper's  
Bones, and  
especially  
by Loga-  
rithms.

together with the other Rods or Bones in Extracting the square or cube Root of any Number given. But the quick Method of Extracting the said Root by the Help of Logarithms, as shall be shewn in the last Chapter of this Treatise. And thus much for the Extracting of the square and cubical Root.

## C H A P. XIII.

*Of Equation, or the Method of solving Questions, by (what is call'd) the Rule of Algebra.*

1.  
*The known  
Quantities  
are to be  
distinguish'd  
from the  
unknown.*

**A**NY Question being propos'd, A, or any other Vowel, to denote the Quantity *sought*, as if it were already known; and put Consonants to denote the Quantities *given*. Or any other like Method may be made Use of, whereby the known Quantities may be distinguish'd from the unknown.

2.  
*An Equation  
is to be form'd.*

Then frame the several Quantities whether given or sought, according to the State of the Question propos'd; out of the said Quantities so fram'd form an Equation; that is, compare the Quantities together, 'till you perceive

some certain of them to be equal to some other, and because in first forming an Equation, the known and unknown Quantities are generally mixt together; therefore the next Thing to be done is, to reduce the Terms of the Equation, as the known or given Quantities may be one Part or Side of it; and the unknown or sought Quantities the other Part or Side. And this is to be done after this Method and Order.

First, If the Quantity sought, or any other thereof be a Fraction, then the Equation is to be reduc'd to another Equation, which may be express'd wholly in Integers. And this is done by turning all the other Terms of the Equation to Fractions of the same Denominator as the Fraction given, and then laying aside the said common Denominator.

Instance: This Equation  $\frac{aa}{4} + 6 = 15$ , amounts to the same as this  $aa + 24 = 60$ .

consequently to the same as this  $aa + 24 = 60$ : Or wholly in Letters thus,  $aa + bc = bd$ , is the same as  $\frac{aa + bc}{b} = \frac{bd}{b}$ , which is the same as  $aa + bc = bd$ .

Secondly, If the known and unknown Quantities are mixt together, transpose the

3.

The known Quantities to be separated from the unknown in an Equation.

4.

First, by reducing Fractions into Integers.

5.

Secondly, by Transposition.

the Quantities under contrary Signs, all the unknown or sought Quantities make up one Side of the Equation, the known or given Quantities the other. Thus,  $aa + 24 = 60$ , will become  $aa = 60 - 24$ ; or  $aa + bc = bd$ , will become  $aa = bd - bc$ . Likewise,  $100 = a$ , will become  $8) 100 = a$ ; or  $Z = 8a$ , will become  $8) Z = a$ .

6.  
Thirdly, by  
Division.

If all the given Quantities happen to be multiplied into any Power of the Quantity sought, then all the Terms must be divided by the lowest Power of the Quantity sought, that is contained in the Equation. Thus,  $3aa = 24$ , becomes  $a = 8$ , namely, by dividing each Side of the Equation by  $aa$ . So  $a^2 + ba^2 = Z^2$  becomes (by dividing each Side by  $a^2$ )  $a + b = Z^2$ .

7.  
Fourthly,  
by Division  
in another  
Case.

But if the highest Power of the Quantity sought, be multiplied into any given Quantity, then all the Terms of the Equation are to be divided by that given Quantity. Thus,  $3aa + 9a = 24$ , becomes  $aa + 3a = 8$ , namely, by dividing each Side of the Equation by 3; or  $baa + d = a$ , becomes  $aa + ba = \frac{a}{b}$ , by dividing each Side by  $b$ .

8.  
Fifthly, by  
reducing

If any Quantity be a *surd* or *irrational* Quantity, then the Equation is to be



ed to another Equation, which may express'd by rational Quantities. Thus,  $x=3$ , will become (by Squaring both Sides of the Equation)  $x=9$ ; or  $x=b$ , will become  $a=bb$ .

Lastly, If only some Power of the Quantity sought makes one Side of the Equation, and one or more Quantities make the other; then the like Power is to be extracted out of each Side or Part of the Equation. Thus,  $x+8=128$ , will become (first by Trans-

position  $\frac{6ax}{5}=120$ ; then by taking  $x$  out of the Fraction,  $6aa=600$ ; then by dividing on each Side by 6,  $aa=100$ ; and lastly, by Extracting on each Side the Square Root)  $a=10$ . Or  $baa+d=f$ ,

become (first  $\frac{baa}{c}=f-d$ , then  $baa=\frac{fc-dc}{b}$ , and lastly)  $aa=\frac{fc-dc}{b}$ .

only remains to be observ'd, that an Equation can be so form'd or ed, as that the known Quantity (or Quantities) can be wholly separated from the unknown, then it is call'd a *simple* or *Equation*. But if after all that can be

fur'd Quan-  
sities into  
Rational.

9.  
Sixthly, by  
Extraction  
of Roots.

III  
Examples  
to be  
done.

10.  
A pure or  
simple E-  
quation,  
what: An  
affected  
Equation,  
what.

be done, the unknown Quantity Quantities) will remain on one affected or mixt with the known, the Equation is call'd an *affected* or Equation. Thus,  $a = \sqrt{\frac{fc-dc}{b}}$ , or  $a^2 = d$ , or  $aa + 2a = 12$ , an affected mixt Equation. It is sufficient to the sign of this Treatise, (this being to young Gentlemen only a Taste of *Algebra*), to subjoin to what has been some Examples of pure Equation, referring such as would go farther in this excellent Art, to my *Latin Treatise of Arithmetick*.

## EXAMPLE I.

II.  
Examples  
of pure  
Equation.

An hundred Pound is to be divided into three Parts, one whereof is to be three Times as much as another, and the Third to be equal to both the first and Two. What will be the three Parts requir'd?

It is obvious from the Tenour of the Question, that any one Part being known, the Rest may be known thereby. Wherefore, let  $a$  denote the least Part as if it were already known. It then follows according to the Tenour of the Question, that the second

be 34; and the third Part, because  
to be equal to the two former, will  
be 34. The several Quantities not given,  
being thus denoted, an (\*) Equation is  
to be found out among the Quantities  
(given or not given) contain'd in the  
Question. And,

According to what was above (*Chap. 11. §. 19.*)  
it shall here be shewn, how this same Question may  
be sol'd by the *Rule of False*, so call'd, because from  
two false Guesses, and their Errors, is deduc'd the  
Quantity sought, by this Rule. Multiply each Guess,  
the Error arising from the other Guess. And if the  
Errors be both alike, (i. e. too much or too little,) di-  
vide the Difference of the Products, by the Difference of  
Errors; but if the Errors be unlike, divide the Sum  
of the Products, by the Sum of the Errors. The Quotient  
will be the Number sought. For Instance: Let 12  
be the Number sought, or least Part in this  
Question. Therefore, the second Part will be  $12 \times 36 = 432$ .  
The third Part  $36 \div 12 = 3$ . Which together make 99,  
as the Error is -4, or 4 too little. Wherefore, let  
the next guess be the Number: Then the second  
Part will be 39, and the third Part 52; which together  
make 91, and so the Error will be +4, or 4 too much.  
Therefore, according to the Rule,

$$\begin{aligned} 1. 12 \times \text{Er. } 2d + 4 &= 48. \text{ Su. of Pr. } 100 \\ 2. 19 \times \text{Er. } 1st - 4 &= 32. \text{ Su. of Er. } 8 \end{aligned} \quad \left( \frac{16}{8} = 2 \right) \text{ 12 N. sought.}$$

by comparing this Operation with the *Algebraical*  
it will evidently appear, the Excellency of the *Algebraical*  
above the *Rule of False*, as to Shortness. And  
the former excels the latter far more as to Exactness,  
in solving all Questions capable of Solution, the  
(whether it be the single or double Rule of *False*)  
in Comparison.

1. Forasmuch as every Whole is equal to all its Parts taken together, thence it follows, that,

$$100 = a + 3a$$

2. Which Equation may be shorten'd, by adding together the several Parts into one Sum; thus,

$$100 = 8a$$

3. And therefore by §. 5, or §. 7, of this Chapter.

$$8) 100 = 12\frac{1}{2}$$

And thus the Quantity of  $a$ , or least Part requir'd is found out.  $8) 100 \text{ l.} = 12\frac{1}{2} \text{ l.}$ , or which is the  $12\frac{1}{2} \text{ l.}$ , that is,  $12 \text{ l. } 10 \text{ s.}$  Wherefore, second Part or  $3a$ , is equal to  $37 \text{ l.}$  And the third Part or  $4a$ , is equal to  $50 \text{ l.}$  Which three Parts put together, make just  $100 \text{ l.}$

## EXAMPLE II.

A Man being ask'd the Age of his Sons, made Answer, that the Second was four Years younger than the First, and the Third was four Years younger than the Second, and the Fourth four Years younger than the Third, withal just Half the Age of the First. What then was the Age of each?



2 denote the Age of the Elder.  
 Therefore,  $a-4$  will denote the Age of  
 Second,  $a-8$  the Age of the Third,  
 $a-12$  the Age of the Fourth. But  
 w, according to the State of the Que-  
 on, the Age of the Fourth, was also  
 if the Age of the Eldest, that is,  
 $a-12=\frac{1}{2}a$ . Whence it is obvious e-  
 nough without any more ado, that  $a=24$ ,  
 that the Age of the Eldest is 24. And  
 the same will be found by Working  
 according to the Rule of Algebra, as fol-  
 lows. Namely,

1. The Age of the  
 Fourth  $a-12$  is, accor- }  
 ding to the State of the }  $a-12=\frac{1}{2}a$   
 Question, Half the Age of }  
 Eldest, that is, }

Wherefore, the said  
 Equation being freed from  
 Fraction by §. 4. of }  $2a-24=a$   
 Chapter, will stand }

Wherefore, by Trans- }  
 ferring 24 under a differ- }  $2a=a+24$   
 ent Sign, according to §. }  
 of this Chapter, }

Wherefore, by Trans- }  
 ferring  $a$  likewise, accor- }  $2a-a=24$   
 ding to the same §. 3. } that is,  $a=24$ .

Having

Having thus found the Age of the eldest Son to be 24, the Age of the second will be 20, of the Third 16, of the Fourth 12, and equal to Half the Age of the Eldest.

### EXAMPLE III.

An *Oxonian* and *Cantabrigian* meet one the other, and comparing by strong each was in Pocket, found thereby, that if the *Oxonian* gave the *Cantabrigian* one Shilling, they should be alike: But if the *Cantabrigian* gave the *Oxonian* one Shilling, the *Oxonian* would have Twice as much as the *Cantabrigian*. It is demanded, how many Shillings each had in his Pocket? Let  $x$  denote the Number of Shillings, which the *Cantabrigian* had. Wherefore,  $x+1$  will denote the Number of Shillings which Both had, after that the *Oxonian* had given one Shilling to the *Cantabrigian*, they being suppos'd in the Question to have had then Both alike. Wherefore,  $x+2$  will denote the Shillings which the *Oxonian* had afore he gave one to the *Cantabrigian*. Wherefore  $x+3$  will denote the *Oxonian's* Money after that he had receiv'd a Shilling from the *Cantabrigian*; and agreeably  $2x$  will denote the *Cantabrigian's* Money after that he had receiv'd a Shilling from the *Oxonian*.

er he had given the *Oxonian* a Shilling. And in this Case, according to the  
 of the Question, the *Oxonian's*  
 money would be Double the *Cantabrigi-  
 a's*; that is,

There would thence }  $x + 5 = 2x - 5$   
 to this Equation, viz. }

Wherefore, by Trans- }  $x + 5 = 2x - 5$   
 position of the single Di- }

And then by Trans- }  $5 = x - 5$   
 moving the  $x$ , which is on }  $5 = x - 5$   
 the Right-hand of the E. }  $5 = x - 5$   
 ation.

being thus found, that the *Cantabrigian* had five Shillings, it will follow,  
 that the *Oxonian* had Seven. Namely,  
 $5 + 1 = 6 = 7 - 1$ . And  $7 + 1 = 8 =$   
 $1$ , that is)  $4 \times 2$ .

#### EXAMPLE IV.

A Captain sends out a third Part of  
 Soldiers, and Ten over, and has still  
 remaining with him Half of the whole  
 Number of his Soldiers, and Fifteen over,  
 demanded, how many Soldiers he  
 in all? Answer 150. Namely, let  $x$   
 denote the whole Number of his Sol-  
 Therefore,  $\frac{1}{3}x + 10$  will denote  
 those

those he sends out. And  $\frac{1}{4}a + 15$  denote those that stay with him. But

1. Those that were sent out, and those that staid with him, made up the whole Number, that is,

$$a = \frac{1}{4}a + 10 + \frac{1}{4}a + 5$$

2. The former Equation being shorten'd, will stand thus:

$$a = \frac{1}{4}a + \frac{1}{4}a + 25$$

3. Wherefore, by taking away the Fractions according to §. 4. of this Chapter,

$$6a = 5a + 150$$

4. Wherefore, by Transposing  $5a$  to the other Side of the Equation.

$$6a - 5a = 150, \text{ that is } a = 150.$$

Namely, 3) 150 (50, and  $50 + 10 = 60$   
And again, 2) 150 (75, and  $75 + 15 = 90$

Which two Sums together make



EXAMPLE V.

A Cistern is supply'd with Water by Pipes, whereof one A will fill the Cistern in  $b$  or 20 Hours, the other B in 30 Hours. In what Time will both together fill the Cistern? *Answer,* 12 Hours. Namely,

Let the Time sought be denoted by

Then by the Rule of Proportion, find how much the Cistern the Pipe A will fill in Time sought viz.

Likewise find, how much of the Cistern the Pipe B will fill in  $a$ , Time sought,

|                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| <i>Ho. Cist.</i> | <i>Ho. Cist.</i> | <i>Ho. Cist.</i> | <i>Ho. Cist.</i> |
| 20               | 1                | $a$              | $\frac{a}{20}$   |
| $20 : 1 :: a :$  | $20$             | $b : 1 :: a :$   | $\frac{a}{30}$   |

|                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| <i>Ho. Cist.</i> | <i>Ho. Cist.</i> | <i>Ho. Cist.</i> | <i>Ho. Cist.</i> |
| 30               | 1                | $a$              | $\frac{a}{30}$   |
| $30 : 1 :: a :$  | $\frac{a}{30}$   | $c : 1 :: a :$   | $\frac{a}{c}$    |

L 3

4. But

4. But according to the State of the Question, the two Portions of the Cistern  $\frac{a}{20}$

and  $\frac{a}{30}$ , or  $\frac{a}{6}$  and  $\frac{a}{2}$ , make up the whole Cistern, that is,

$$\frac{a}{20} + \frac{a}{30} = 1$$

$$\frac{a}{6} + \frac{a}{2} = 1$$

5. Which Equation, its Fractions being reduc'd to the same Denomination, will be

$$\frac{30a \times 20a}{600} = 1$$

$$\frac{ba \times ca}{60} = 1$$

6. Wherefore, by Transposition or equal Multiplication, viz. by 600 or  $bc$ ,

$$30a \times 20a = 600$$

$$ba \times ca = 60$$

7. And then by equal Division, viz. by  $(30 + 20 =) 50$ , or  $b + c$

$$a = \frac{600}{50} = 12$$

$$a = \frac{60}{b+c}$$

That  $\frac{bc}{b+c}$  or 12 is the true Number sought may be prov'd by solving this following Question

being the Inverse of the former, viz.  
 with the Pipes A and B fill the Cistern in  
 or 12 Hours; and the Pipe A alone  
 fills it in  $c$  or 20 Hours: In what Time  
 therefore will the Pipe B alone fill it?  
*Answer*, In 30 Hours. For let the Time  
 ought be  $a$ . Therefore,  $a : b :: b : c$   
 and  $c : 1 :: b : bc$ . But according to the  
 state of the Question,  $\frac{b}{a} + \frac{b}{c} = 1$ . Where-  
 fore, by *Reduction*,  $a = \frac{bc}{c-b} = 30$ . So the  
 solution of both Questions agree one  
 with the other, and consequently, both  
 are truly solved.

### EXAMPLE VI.

Three Companies of Soldiers passing by,  
 the First takes away from a Shepherd  
 half of his whole Flock, and Half of a  
 single Sheep over. The Second takes  
 away Half of the Remainder of the  
 flock, and also Half a Sheep over. The  
 third likewise takes away Half of the  
 flock yet remaining, and Half a Sheep  
 over. All which was done without kil-  
 ling any Sheep, and there remain'd at  
 last  $b$ , or twenty Sheep to the Shepherd.  
 How many Sheep therefore had he in his  
 flock at the First? *Ans*, 167. Namely,

L 4

r. Let



1. Let the Number  
of Sheep at the first  
be

2. Therefore, the  
Number of Sheep ta-  
ken by the first Com-  
pany, will be

3. And so the first  
Remainder left to the  
Shepherd, will be

4. Then the Half  
of the first Remainder  
is  $\frac{1}{2}a - \frac{1}{2}$ , to which  
adding ( $\frac{1}{2}$ , or which  
is the same)  $\frac{1}{2}$  of a  
single Sheep, the Sum  
will be the Quantity  
taken by the second  
Company, viz.

5. Wherefore, the  
second Remainder  
left to the Shepherd,  
will be ( $\frac{1}{2}a - \frac{1}{2}$ , or  
which is the same)  
 $\frac{1}{2}a - \frac{1}{2}$  less  $\frac{1}{2}a + \frac{1}{2}$ ,  
that is,



6. Acc



Accordingly, Half

the second Re-

nder will be

$\frac{1}{2}$ , to which ad-

$\frac{1}{2}$ , or which is

(the same)  $\frac{1}{2}$  of a fin-

Sheep, the Sum

be what was ta-

away by the

Company, viz.

Wherefore, the

Remainder left

the Shepherd, will

$\frac{1}{2}a - \frac{1}{2}$ , or which

(the same)  $\frac{1}{2}a - \frac{1}{2}$

$\frac{1}{2}a + \frac{1}{2}$ , that is,

But this third

remainder is accor-

to the State of

Question, equal

or 20, that is,

Wherefore, by

disposition

o. Wherefore, by

ally multiplying

Side of the E-

tion by 8, there

arise the Num-

ought, viz.

$$\frac{1}{2}a + \frac{1}{2}$$

$$\frac{1}{2}a - \frac{1}{2}$$

$$\frac{1}{2}a - \frac{1}{2} = b = 20.$$

$$\frac{1}{2}a = b + \frac{1}{2} = 20 + \frac{1}{2}$$

$$a = 8b + 7 = 167.$$

Accor-

Accordingly, the Half of 167 is 83½, to which adding  $\frac{1}{2}$ , the Sum is 84, Number of Sheep taken by the first Company; and then the first Remainder 83. Again, the Half of 83 is 41½, which adding  $\frac{1}{2}$ , the Sum is 42, Number of Sheep taken by the second Company; and so the second Remainder was 41 Sheep. Lastly, the Half of 41 is 20½, to which adding  $\frac{1}{2}$ , the Sum is 21, the Number of Sheep taken by the third Company; and so the third Remainder is 20 Sheep according to State of the Question.

## C H A P. XIV.

### *Of the Use of Logarithms.*

1.  
Logarithms of  
great Use  
in Multi-  
plication,  
Division,  
and the  
Extraction  
of Roots.

2.  
Logarithms,  
what.

**I**T has been above observ'd, that *Multiplication and Division* of Numbers, as also the Extraction of Root of any Power, is render'd more easy by the Help of *Logarithms*. I have therefore reserv'd this last Chapter to explain therein the Use of *Logarithms*.

*Logarithms* are artificial Numbers, proceeding in *Arithmetical* Proportion the natural Numbers, to which the

ed, do proceed in Geometrical Pro-  
 gression, viz.

Numbers. Logarithms.

|      |         |
|------|---------|
| 1    | 0.00000 |
| 10   | 1.00000 |
| 100  | 2.00000 |
| 1000 | 3.00000 |
| &c.  | &c.     |

Every Number consists of two Parts, Numerative and Denominative; so every Logarithm consists of two Members, distinguish'd one from the other by some separating Mark, whether full or Comma, &c. That Member of Logarithm, which stands on the left-hand of the separating Mark, has respect to the numerative Part of the Number, to which the Logarithm belongs; the other Member on the Left-hand, has respect to the denominative of the said Number. Namely, here the Denomination or Place of the last (left-hand) Figure, and consequently of all the other Figures in the Number is indicated or shewn; whence this Member of the Logarithm is peculiarly call'd the (\*) Index. For Instance: The

3  
 Every Logarithm consists of two Parts

It is otherwise call'd the Characteristic

Index

*Index* [0] being affix'd to a *Logarithm* denotes, that the last Figure of the Number, to which the *Logarithm* answers, nothing distant from (*i. e.* is in) the Place of Units, and consequently, that said Number is one of the Digits. *Index* (1) denotes the last Figure of correspondent Number, to be distant Place from the Place of Units, *i. e.* be in the Place of Tens; and consequently, the Number it self to be either Ten, or some Number between 10 and 100. And so of other *Indices*.

4.  
How the Agreement of Logarithms, answers to the Agreement of Numbers.

Hence all Numbers, which have the same Denominative, but not the same numerative Part, (such as are all Numbers from 1 to 10, or from 10 to 100, &c.) will likewise have *Logarithms* whose *Indices* will be the same, but not their other Members. And on the other Side, all Numbers, which have the same numerative Part, but not Denominative will also have the same *Logarithm*, excepting only the different *Index*. Instance:

|                                                               |                                      |
|---------------------------------------------------------------|--------------------------------------|
| Numb. of same Denominative Value, but not of same Numerative. | Their respective <i>Logarithms</i> . |
|---------------------------------------------------------------|--------------------------------------|

|     |        |
|-----|--------|
| 256 | 2.4082 |
| 257 | 2.4099 |
| 258 | 2.4116 |

Nu



Num. of same Nu- Their respective  
merative, but not de- Logarithms.  
minative Value.

|      |         |
|------|---------|
| 256  | 2.40824 |
| 25L6 | 1.40824 |
| 2L56 | 0.40824 |

If a Number be purely or wholly a  
Decimal, then to the *Logarithm* thereof  
to be affixt a *negative Index*, shewing  
Distance of its first (i. e. left-hand)  
significant Figure from the Place of U-

Thus, the *Logarithm* of the Deci-  
mal 256, is 1.40824; the *Logarithm*  
of the Decimal 0256, is 2.40824, &c.  
*Logarithms* are of great Use in multi-

plying or dividing larger Numbers; for-  
example, such as the *Logarithms* of the two  
Factors in *Multiplication*, being added to-  
gether, make up the *Logarithm* of the  
Product; and in *Division*, the *Logarithm*  
of the Divisor, being subtracted from  
the *Logarithm* of the Dividend, leaves  
the *Logarithm* of the Quotient. For  
Example:

5.  
Of the Lo-  
garithms  
of Deci-  
mals.

6.  
Multipli-  
cation and  
Division,  
how shor-  
ten'd by  
Loga-  
rithms.

| Num. Logarithms. | Numbers.     | Logarithms. |
|------------------|--------------|-------------|
| 816              | 816          | 2.91168     |
| + 1.07918        | divide by 12 | - 1.07918   |
| <hr/>            |              | <hr/>       |
| 2.91168          | 68           | 1.83250     |
| <hr/>            |              | <hr/>       |

In

7.  
The making of any  
Power, and the  
Extraction of any  
Root; how shorten'd  
by Logarithms.

In like manner, the *Logarithm* of a *Root*, being multiplied into the *Index* of any *Power*, gives the *Logarithm* of the said *Power*: That is, the *Logarithm* of a *Root*, being doubled, gives the *Logarithm* of the *Square*; being triplicated gives the *Logarithm* of the *Cube*. So on. Thus, because the *Logarithm* of 9, is 0.95424; therefore, 0.95424  $\times$  2 = 1.90848, the *Logarithm* of 99 or 81. And 0.95424  $\times$  3 = 2.86272, the *Logarithm* of 99 or 729. Namely,

|                                      |                                      |            |                                      |
|--------------------------------------|--------------------------------------|------------|--------------------------------------|
| 9                                    | 0.95424                              | 9          | 0.95424                              |
| $\times 2$                           | + 0.95424                            | $\times 2$ | 0.95424                              |
| <hr style="width: 50%; margin: 0;"/> | <hr style="width: 50%; margin: 0;"/> | $\times 2$ | <hr style="width: 50%; margin: 0;"/> |
| Squ. 81                              | 1.90848                              |            | <hr style="width: 50%; margin: 0;"/> |
|                                      |                                      | Cube 729   | 2.86272                              |

And therefore, on the other Hand, the *Logarithm* of any *Power*, being divided by the *Index* of the said *Power*, give the *Logarithm* of the *Root*. For Instance: The *Logarithm* of the *Square* is 1.90848. Wherefore, 2) 1.90848 (0.95424, the *Logarithm* of 9. In like manner, the *Logarithm* of the *Cube* is 2.86272. Wherefore, 3) 2.86272 (0.95424. Only here it is to be noted that although in common *Division* of *Integers*, when the *Divisor* is bigger than the left-hand Figure of the *Dividend*

Cypher is omitted in the Quotient, being of no Use in a Quotient of Integers; yet here in this Sort of *Division Logarithms*, (as also in the *Division Decimals*;) the Cypher is not to be put in the Quotient, because it is the *Index* of the *Logarithm*.

In other Respects, the *Addition*, *Subtraction*, *Multiplication*, and *Division* of *Logarithms*, differ not at all from the Operations of *Integers*, if the *Indices* of the *Logarithms* be affirmative, *i. e.* (if the Numbers, to which the *Logarithms* refer, be *Integers*. But if the *Indices* be all or some of them) negative, *i. e.* the Numbers, to which the *Logarithms* refer, be (all or some of them) *Decimals*; then in Working the *Indices*, the Algebraical Rules concerning the Signs and —, are to be observ'd. For in the *Logarithm* of a decimal Fraction, the *Index* alone is to be esteem'd Negative, the other Member is to be esteem'd affirmative. And therefore, if any Thing be carried from the affirmative *Index*, to the negative *Index*, that (being it self affirmative) is to be subtracted with the *Index*, as Affirmatives and Negatives are wont to be compar'd together. Thus, in the first Example of *Addition*, where the *Indices* are compar'd together,  $1 + \bar{1} = 0$ .

And

8.

Of working affirmative and negative *Indices* together, or negative *Indices* alone.

And in the second Example of Addition, where 1 is carried from the affirmative Members, to the Indices 1 and 3, negative Index 1 of the Sum arises, viz.  $1 + 3 + 1 = 1$ . Also in the second Example of Multiplication, where Affirmative 2 is carried to the Product  $2 \times 5$ , the negative Index 8 of the Product arises, thus, viz.  $2 \times 5 = 10$ ,  $10 + 2 = 8$ . Hence, that Division undo, what Multiplication has done, Rule is to be follow'd in the Division of negative Indices. Namely,

9. A Rule concerning the Division of negative Indices.

If the Divisor does exactly measure the negative Index, then the Number found by Division is the Quotient. otherwise the negative Index is to be increas'd by Units, 'till the Divisor exactly measure it; and the Number thence arising by Division is the Quotient; and what was added to the negative Index, (which is always Affirmative,) together with the next Figure of the Dividend makes the next Dividend. Thus, in the second Example before Division, because 5 will not exactly measure 8, therefore 8 is to be increased to 10, and the Affirmative 2, added to make it 10, together with 3, the Figure of the Dividend, is to constitute the next Dividend 23. Come we



illustrate the several Cases that may  
en, by the Examples following.

**Examples of Addition.**

| Log.    | Numb. | Log.    | Numb.   | Log.    |
|---------|-------|---------|---------|---------|
| 1.83250 | 73    | 1.86332 | L061    | 2.78532 |
| 1.07918 | *L005 | 3.69897 | *L004   | 3.60206 |
| 0.91168 | L365  | 1.56229 | L000244 | 4.38738 |

**Examples of Subtraction.**

| Log.    | Numb. | Log.    | Numb.   | Log.    |
|---------|-------|---------|---------|---------|
| 0.91168 | L365  | 1.56229 | L000244 | 4.38738 |
| 1.07918 | .73   | 1.86332 | .L004   | 3.60206 |
| 1.83250 | L005  | 3.69897 | L061    | 4.78532 |

**Examples of Multiplication.**

| Numb.                      | Log.                   |
|----------------------------|------------------------|
| of L029 = 000841.          | 2.46239 * 2 = 4.92478. |
| Cub. of L03 = L0000000243. | 3.47712 * 5 = 8.38560. |

**Examples of Division.**

| Numb.             | Log.                 |
|-------------------|----------------------|
| 000841 = 029.     | 2) 4.92478 (2.46239. |
| L0000000243 = 03. | 5) 8.38560 (1.67712. |

10. It remains now to shew the Use of the Canon or Table of Logarithms, which is two-fold, either having any Number given to find its Logarithm, or having any Logarithm given to find its Number.

11. If the Number given be an Integer, and under 100, having found the Number in some Column of the Table which has N on the Top of it, its respective Logarithm is plac'd on its Right hand in the adjoining Column of the Table, together with the proper Index or Characteristic belonging thereto. Thus, the Logarithm of 4, is 0.60206. The Logarithm of 24, is 1.38021.

12. If the Integer given be 100, or any Number above to 1000, then having found the same likewise in some Column of N, its respective Logarithm is plac'd also in the adjoining Column, all the Index, which (according to what was observ'd, §. 3.) is 2. Thus, the Logarithm of 112, is 2.049218.

13. If any Integer from 1000 to 10000 given, then you must find such a Number in the Column N, as added to the top Figures, will together make up the Number given. And under said top Figure, and opposite to the Number in the Column N, will be

*Logarithm* sought. Thus, supposing  
we requir'd to find the *Logarithm* of  
1012; I find 101 in the Column N,  
with the top Figure 2, makes to-  
ter 1012. And consequently, the  
nd *Logarithm* under the top Figure  
nd which is opposite to 101, is the  
*Logarithm* sought, viz. 005181, to which  
cause the left-hand Figure of 1012,  
the third Place from the place of  
is to be affixt the *Index* 3; and  
the entire *Logarithm* of 1012, is  
05181.

the Number, whose *Logarithm* is  
nt, be partly an *Integer*, partly a  
mal; or if it be purely and wholly  
decimal, then look for its *Logarithm*,  
it had been wholly an *Integer*;  
ging or putting such an *Index* to the  
*Logarithm* thus found, as answers to the  
ber given. Thus, the *Logarithm* of  
will be 1.563481. The *Loga*-  
of 366, will be 0.563481. The  
*Logarithm* of 366, will be 1.56348.  
As for common Fractions, it is  
to turn them into Decimals, if you  
d work by *Logarithms*; and then  
nd the *Logarithm* of the decimal  
ion, answering to the common Fra-  
given.

14.  
To find  
the *Loga*  
rithm of  
a Deci-  
mal, or a  
Number  
partly In-  
teger, and  
partly De-  
cimal.

nd 2

if

15.

To find  
the Num-  
ber an-  
swering to  
any Loga-  
rithm gi-  
ven.

If you would find the Number answering to any *Logarithm* given, without any Regard had to the *Index*, the other Part of the *Logarithm* given, and the Number answering thereto, the Number sought; which is such *Integer* or *Decimal*, as the *Index* of *Logarithm* given shews. Thus,

|          |                                          |      |
|----------|------------------------------------------|------|
| 1.079181 | } Is the <i>Lo-</i><br><i>garithm</i> of | 12   |
| 2.079181 |                                          | 120  |
| 3.079181 |                                          | 1200 |
| 1.079181 |                                          | 12   |
| 2.079181 |                                          | 120  |
| 3.079181 |                                          | 1200 |
| Etc.     |                                          |      |

16.

Further,  
in refe-  
rence to  
the same.

If you can't find in the Table the same *Logarithm* as is given, but find therein one that differs from it in the Right-hand, or the two Right-hand Figures; then you may take the same for the *Logarithm* given, and the Number answering thereto, as the Number sought. And accordingly, if you have added two *Logarithms* together, the Sum or *Logarithm* arising thence can't be exactly found in the Table, you are not presently to conclude, that you have not added the *garithms* given rightly together; but



if you can't find some *Logarithm* in Table, that differs from the *Logarithm* arising by *Addition*, only in one of the right-hand Figures; and so, you may take the same for the *Logarithm* requir'd, as if it did exactly agree with the *Logarithm* arising from *Addition*. And the like is to be understood as to *Subtraction*, &c.

It remains only to observe, that the use of *Logarithms* most frequently occurring in *Trigonometry*, therefore the use or Table of *Logarithms* belonging to this Course of *Mathematicks*, is join'd to the Treatise of *Trigonometry*, belonging likewise to this Course of *Mathematicks*. And the said Table is continued no further than 10000, not only because this is sufficient for common calculations, but also, because it could be continued further without swelling it to too great a Bulk, but by what is call'd) a Table of proportional parts; which renders multiplying and dividing by *Logarithms* little (if at all) troublesome and tedious, than by the common Way, especially with the use of *Naper's Bones*.

17.

The Table of *Logarithms* belonging to this Course of *Mathematicks*, where to be found.

18.  
The Con-  
clusion.

And thus I have here laid together so much of *Arithmetick*, as seems requisite for *Young Gentlemen* to know. Such as are desirous to have a farther Insight into the Grounds, also more critical or curious Parts of *Arithmetick*, I refer them to my *Treatise of Arithmetick*, entitled, *menta Arithmetice Numeroſe & Spec in uſum Juuentutis Academicæ.*

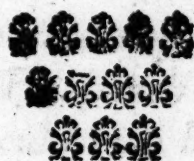
**F I N I S.**

2.

THE  
Young Gentleman's  
GEOMETRY,

Containing such  
ELEMENTS of Geometry,  
as are most useful and easy to be  
known.

By EDWARD WELLS, D. D.  
Rector of Cotesbach in Leicester-  
shire.



L O N D O N,  
Printed for James Knapton, at the Crown  
in St. Paul's Church-Yard. 1713.

8

Young Gentlemen's

GEOMETRY

Containing such  
THE MEANS of  
as are most useful and easy to be  
known.

EDWARD HULL, D.D.  
Rector of St. Andrew's Church in London.

LONDON  
Printed for James Knapton, at the Crown  
in St. Paul's Church-Yard, 1713.



# THE PREFACE.

GEOMETRY is of excellent

Use, consider'd only, as it  
does to accustom the Mind to close  
true Reasoning. But its Excel-

lency is still greater, if we consider  
its extensive Usefulness, both in the  
publick and private Affairs of Life.

Hence it were much to be wish'd,  
that Young Gentlemen were as ge-

nerally taught the Elements of Geo-  
metry, as they are the Rudiments  
of Grammar: Forasmuch as the Ad-

vantage they would reap by Learning  
the former, will upon a fair Com-  
parison be found to be no less, not

to say more, than those they acquire  
by the latter.

Upon due Enquiry it will (I be-  
lieve) appear, that Nothing has done  
more

## The Preface.

more Disservice in this Point, more discourag'd Young Gentlemen from Venturing upon Geometry, than the Notion, that a competent Knowledge of such Geometrical Elements as are of most Use in the common Concerns of Life, can't be attain'd without extraordinary Pains and Time. And this Notion seems to owe its Rise, partly to an Opinion that all Euclid's Elements are necessarily to be understood, in order to attain such a Knowledge; and partly to the voluminous Bulk, wherein Euclid has been sent abroad into the World by some Commentators.

To remove therefore this wrong Notion, I have in the following Treatise reduced (I think, I may say) not if not all those Elements of Geometry, that are of greatest Use in the common Concerns of Life, and requisite to be known by Young Gentlemen, under twenty-eight principal Propositions, viz. twelve Theorems, and twelve Problems relating

## The Preface.

Lines and Planes, and four Theorems relating to Solids.

And to render the Going through these few Propositions (with their Corollaries) more Entertaining and pleasant, I have not only subjoin'd to several Theorems (and their Corollaries) relating to Lines and Planes, the Use of their Uses, but have also illustrated the same by Examples, relating to taking Heights and Distances, or to Surveying, &c. The Problems relating to Lines and Surfaces, carry Evidence enough of their Usefulness in their Operations. And for the Theorems relating to Solids, they also carry Evidence enough of themselves of their Usefulness in the Measuring of Solids or Bodies, Vessels of Capacity.

Wherefore, the Length and Dryness of the Study of Geometry, at least so far forth as is most requisite for Young Gentlemen, being thus remov'd, there remains nothing that, with any Colour of Reason, discourages

## The Preface.

courage them from Going through  
short Treatise of Geometry, and  
Attaining a competent Knowledge  
the most useful Elements of Geo-  
try ; especially, if they are but  
any tolerable Degree animated (as  
ought) with a Desire of Know-  
what may be of great Use to the  
both in the publick and private  
Affairs of Life ; and so render the  
the more capable of promoting  
Welfare, either of their own Fa-  
lies in particular, or of their Co-  
try in general.

THE



# THE Young Gentleman's GEOMETRY.

## The INTRODUCTION.

**T**HE word *Geometry*, in its Original, or the *Greek* Tongue, do's literally import no more than *the Measuring of Land*, or *the Earth*. And it seems to be taken the whole Art or Science of *Measuring* *tinuous Quantity*, or *Magnitude* in general; either, because the primary and principal End of cultivating this Art was *measure Land*, or else, because *the Earth is the principal Magnitude*, or *greatest Body*, that can actually be measur'd. And forasmuch as *Magnitude* has three mentions, (*i. e.* may be measur'd three manner of Ways, *viz.* as to its) *Length*, *Breadth*, and *Depth*; and consequently may be consider'd, either as to *Length* only

1.  
Geometry;  
what, and  
why so  
call'd.

2.  
Geometry  
distinguish-  
able into  
three ge-  
neral  
Parts.

only, under which Consideration it is call'd a *Line*; or else, as to Length and Breadth together, under which Consideration it is call'd a *Surface*; or lastly, to all its three Dimensions jointly, under which Consideration it is call'd a *Solid Body*: Hence *Geometry* is distinguish'd into three general Parts, whereof the first treats of *Lines*, the second of *Surfaces* and the third of *Solids* or *Bodies*.

3.

The general Method follow'd in this Treatise.

But forasmuch as *Lines* and *Surfaces* have such a mutual Dependence, as they may best be treated of together, and forasmuch as it will be sufficient to the Design of this Treatise, to speak only of plain *Surfaces*, I shall, therefore, in the three Chapters which make up this Geometrical Tract, follow this general Method. In the first shall be contain'd the most useful and easy *Theorems*, relating to *Lines* and plain *Surfaces*: In the second shall be contain'd the most useful and easy *Problems*, relating likewise to *Lines* and plain *Surfaces*: And in the third and last shall be contain'd what relates to *Solids*, so far forth as is agreeable to the Design of this Treatise.

4.

A Geometrical Proposition, Theorem,

Now, as by a *Proposition*, *Mathematicians* denote in general somewhat propos'd, whether it be to be demonstrat

Problem,  
Corollary,  
and Lem-  
ma, what.

made, so by a (\*) *Theorem*, they un-  
derstand in particular somewhat propos'd  
to be demonstrated or proved, (as  
all the Angles in a Triangle are e-  
qual to two Rights) and by a (+) *Problem*,  
they particularly understand somewhat  
propos'd to be made or done, as to make  
an equilateral Triangle, &c. Whatever is  
deriv'd or follows from a Demonstration,  
hence call'd a *Consequence*: And be-  
cause the same may be look'd upon as  
somewhat, gain'd from the said Demon-  
stration, over and above what it was pri-  
marily intended to prove, hence it is de-  
noted by the more usual Name of a (||)  
*Corollary*. Any Demonstration, made  
not only on account of its being ser-  
viceable to some other following Demon-  
stration, is peculiarly stil'd a (\*) *Lemma*.  
Lastly, A (+) *Scholium* is an Annotation

It properly denotes in the Greek Tongue somewhat  
added on, and hence it is taken here to denote spe-  
cial or contemplative Truths.

In the Greek Tongue it denotes no more than a  
propos'd, and so is restrain'd to this Geometrical  
application only by Use.

The Word does properly signify a *Coronet*, or some-  
thing given to Actors, Fencers, &c. over and above  
their due Stipends and Wages.

The Word signifies in the Greek Tongue somewhat  
added, and so it is us'd by *Mathematicians* to denote a  
Demonstration taken only to prove another.

It is likewise a Word of Greek Original, us'd to sig-  
nify a Note or Annotation.

or

or Remark, made in reference to Theorem or Problem. As that the Theorem may be demonstrated, and the Problem work'd some other Way, &c.

5.  
An Axiom,  
Definitio-  
on, and  
Postula-  
tum, what.

There are also us'd, by *Mathematicians* in their Geometrical Treatises, *Axioms* and *Definitions*; from which they deduce the Demonstration of their Propositions primarily and originally. For an (||) *Axiom* is denoted a Self-evident Truth, which therefore is worthy to be receiv'd or assented to, without Proof. And by a (\*) *Definition* is understood an Explication of the Sense Meaning, wherein *Mathematicians* use Word. Besides these, some *Mathematicians* expressly set down *Postulates*, which are denoted Self-evident Problems. *i. e.* Things so plain and easy to be done, as, without Proof, to require Assent, that they may be done; as Instance, that a *right Line* may be drawn from a Point to a Point given, &c.

(||) It denotes in the *Greek* Tongue somewhat Weak, and so is taken by *Mathematicians* to denote a Proposition worthy to be assented to without Proof.

(\*) It is so call'd, because as Lands are distinguished one from the other by their respective (*Fines*) Boundaries, so Words by their respective Definitions,



CHAPTER I.

Containing the more useful and easy Theorems, which relate to Lines and plain Surfaces. To which are premis'd such Definitions and Axioms, as are requisite to the Demonstration of the said Theorems.

(\*) Definitions.

Mathematical (+) *Point* is conceiv'd to have neither Length nor Breadth, Depth, and consequently to have no *Def. 1.*  
*what.*

Mathematical (||) *Line* is conceiv'd to have only Length, no Breadth nor Depth. *Def. 2.*  
*what.*

Definitions are plac'd by *Mathematicians* before, because some Words, that occur in the Axioms, it to be defin'd or explain'd.

A *Point* is said to be conceiv'd, such as it is here, because it can't *actually* be describ'd so. It is denoted by one Letter.

A *Line* is said likewise to be conceiv'd, such as it is defin'd, because it can't *actually* be so describ'd.

A *Line* does not exist alone in Nature, yet its use is useful to be known, because all Distances consider'd only as right Lines, i. e. in measuring the of two Objects, we consider only Length, not nor Depth. A *Line* is generally denoted by two, one being plac'd at each End of it, as *Fig. 1.* A *Line*, nor joining to any other, is sometime denoted by single Letter.

N

For

For it is conceiv'd to be describ'd by Flux or Motion of a Point.  $\circ$  Thus, 1. the Line AB is conceiv'd to be scrib'd by the Motion of the Point from A to B.

Def. 3.  
A right,  
and curve  
Line,  
what.

If the Line AB lies all along its Length exactly even with (i. e. is no way higher or lower, more to one or other Side, than) the two Points A & B, which bound it; then it is call'd a *right* (or *straight*) Line. Otherwise it is call'd a *curve* (or *crooked*) Line; the Line CD, Fig. 2. of which the Side CED is call'd the *concave* Side, and Side CFD the *Convex*.

Def. 4.  
A Super-  
ficies or  
Surface,  
what.

A mathematical (\*) *Superficies* or *Surface*, is conceiv'd to have Length & Breadth, but no Depth. For it is conceiv'd to be describ'd by the Motion of a Line. Thus, Fig. 3. the Surface ABCD, is conceiv'd to be describ'd by the Motion of the Line AB, from A to CD.

Def. 5.  
A Plane,  
what.

If the Surface ABCD lies every way exactly even with the Lines AB and CD which bound it; then it is call'd a *Plane*.

---

(\*) A *Superficies* or *Surface* is said likewise to be conceiv'd, such as it is here defin'd, because it can't be otherwise describ'd. And although no *Superficies* do's exist alone in Nature, yet in Measuring of Land, or Pavement (&c.) it is the *Superficies* of them, which is alone consider'd. And therefore the Knowledge of the Affections of *Surfaces* is of great Use.

Surf

face, and in one Word, a (+) *Plane*.  
 otherwise it is call'd a *curve Surface*,  
 which being consider'd as to its rising  
 side, is call'd a *convex Surface*; being  
 consider'd as to its other Side, is call'd a  
*concave Surface*.

The mathematical Term (||) or *Bound* Def. 6.  
 any Thing is its Extremity, or End. A Term,  
 thus, the two Terms of the Line AB, what.  
 (Fig. 1.) are its two extream Points  
 and B. And the Terms of the Surface  
 CD, (Fig. 3.) are the four Lines AB,  
 DC, and CA.

A mathematical *Figure* is (\*) compre- Def. 7.  
 ended (i. e. enclos'd all round) with A Figure,  
 one or more Terms or Bounds. what.

A Superficies or Surface, if it be confi- Def. 8.  
 d as (+) terminated or bounded, A plain  
 makes a *superficial Figure*; which, if its Figure,  
 face be plain, is call'd a *plain* (or what.

It being usual to make the Draughts of Cities or  
 &c. upon plain Surfaces or Planes, hence (by a fi-  
 gure Way of speaking) a *Plane*, or as it is writ and  
 us'd by the French a *Plan*, is taken to denote the  
 draught made upon it. Thus, the *Plan* of a Fort, is the  
 draught of a Fort, made upon the said Plan or Plane.

This Definition is requisite to be taken Notice of,  
 in common Speech, by the Term or Bound of a  
 Thing is often meant, not its own Extremity, but some  
 other Body. Thus, we say an *Island* is bounded all  
 round by the Sea.

Hence two right Lines can't make a Figure.  
 For sometimes in Mathematicks it is so; so con-

plane) Figure. And such only are spoke of in Chap. 1, and 2, of this Geometrical Treatise.

Def. 9.

A Circle,  
its Cir-  
cumfe-  
rence, and  
Rays,  
what.

A mathematical  $\textcircled{H}$  Circle is a plane Figure, comprehended under one curved Line, call'd the *Circumference*. The middle Point of a Circle is call'd its *Center* from which the Circumference is equally distant all round. Whence right Lines drawn from the Center to the Circumference, and call'd the *Rays* of the Circle are all equal one to another. Thus, Fig. 4. C is the Center, ABDE the Circumference, the Lines CA, CB, CD, and CE are so many Rays of the Circle CABDE.

Def. 10.

A Diameter,  
and  
Semi-circle,  
what.

A right Line AD (passing through the Center C, and bounded at each End A and D by the Circumference; or, what comes to the same) made up of (\*) Rays AC and CD, is call'd a (†) Diameter.

(||) It is to be well observ'd, that it hence appears, *Mathematicians* use the Word *Circle* in a different sense from that, wherein it is commonly us'd. In common Use, a *Circle*, generally denotes the *circular* or *curved* Line, call'd by *Mathematicians* the *Circumference*; whereas in *Geometry*, a *Circle*, denotes not a *circular Line*, but a *circular Plane*, or a Plane bounded by a circular Line.

(\*) A Diameter being thus made up of two Rays, hence, a Ray is otherwise call'd a *Semi-diameter*.

(†) Because the two Points A and D of the Circumference, which bound the Diameter, are the most opposite Points in the Circumference; hence, two Things, which have the greatest Opposition one to the other, are said to be *diametrically opposite*.



er, and divides the Circle into two  
nal Parts, thence call'd *Semi-circles* ;  
are the Figures CABD, and CAED,

4.  
If the right Line ST (Fig. 5.) be boun-  
at each End S and T by the Cir-  
ference, and (is not made up of  
Rays, or, which comes to the same)  
les not through C the Center ; then  
Figure contain'd by the right Line  
and the Part of the Circumference  
T, is call'd a *Segment* of the Circle  
TGM, viz. SGT the lesser Segment,  
SMT the greater. And the right  
e ST is call'd the *Base* of the said  
ments. The Figure ACB is call'd the  
or of a Circle.

Def. 11.  
A Seg-  
ment and  
Sector of a  
Circle,  
what.

Any Part of the Circumference is call'd  
*Arch* of the Circle. Thus, Fig. 4.  
BD, DE, and EA, are four Arches  
the Circle CABDE. And likewise  
OT, TM, and MS, are so many  
hes of the Circles CSGTM, Fig. 5.

Def. 12.  
An Arch  
of a Circle,  
what.

The whole Circumference of every  
le is divided by *Mathematicians* into  
(II) equal Parts, call'd *Degrees* ; and

Def. 13.  
A Degree,  
Minute,  
(&c.) of  
a Circle,  
what.

As all the Degrees of the same Circle are equal one  
other, so the Degrees of all Circles are *proportional*,  
every Degree of a lesser, as well as of a greater Cir-  
a 360th Part of its own Circle. The Reason of  
ing all Circles into 360 Degrees, is probably sup-  
to be, because this Number contains many (aliquot  
i. e.) Parts, which will exactly divide it.

N 3

each

each Degree into 60 *Minutes*, and each Minute into 60 *Seconds*, and each Second into 60 *Thirds*, &c.

**Def. 14.**

An Angle,  
the Legs  
and Head  
of an An-  
gle, what.

The mutual (\*) Inclination of Lines meeting together, is call'd an *Angle*. And the Lines thus meeting together, are call'd the *Legs* of the Angle. And the Point, wherein they meet, call'd the *Vertex* or *Head* of the Angle, or the *angular Point*. Thus, *Fig. 4.* Lines AC and BC, are the Legs of Angle (+) ACB; the Lines BC and CD are the Legs of the Angle (+) BCD; the Vertex or Head of each Angle is the Point wherein their respective Legs meet.

(\*) Some define an *Angle* to be the *Aperture* (or *Gap*) between the two *Legs*; and illustrate the various Angles, by the various Aperture of the Legs of a Pair of Compasses. But, since it is evident, that the Inclination of the Compasses Legs, vary according to the various Aperture, it follows, that both Definitions come to the same. And therefore I have retain'd the old Definition of *Euclid*, only omitting that Restriction, that the Lines must not lie *directly* one with the other: Forasmuch as this is included in the Word *Inclination*, according to our common Acceptation of it.

(+) An Angle is generally denoted by three Letters, which the middle One denotes the Head of the Angle, as in the Angles ACB, and BCD. But sometimes an Angle is denoted only by a single Letter, *viz.* when there is but one single Angle about the Point, and so it can't be mistaken, what Angle is meant; as the Angle A, in *Fig. 9.* and 10.

If the Legs of the Angle meet together in the same Plane, then it is call'd a Plain, or) *Plane-angle*. And if the Legs be also right Lines, then 'tis call'd a *Linear* or *right-lin'd Angle*. And it is to be noted, that all the Angles spoken of in *Chap* 1, and 2, of this Treatise, are *right-lined Plane-angles*.

Every such Angle is measur'd by the Arch of a Circle, whose (||) Center is the Head of the Angle. For that Arch of the Circle, which is intercepted between the Legs of the Angle, is the Measure of the Angle, i. e. the Angle is said to be an Angle of so many Degrees, as are contain'd in the Arch. Thus, the Measure of the Angle ACB, (*Fig. 4*, or *Fig. 5*) is the Arch AB; which being a 4th Part of the Circumference, and so containing 90 Degrees, therefore the Angle ACB, is said to be an Angle of 90 Degrees.

An Angle of 90 Degrees is call'd a *Right Angle*, any other is call'd in general an *oblique Angle*: And if the oblique

**Def. 15.**  
A Plain, and right-lin'd Angle, what.

**Def. 16.**  
The Measure of an Angle, what.

**Def. 17.**  
A right, oblique, acute, and obtuse Angle, what.

The Circle may be drawn at any Extent from the Head of the Angle. For the Legs of the same Angle will intercept equal Arches in any Circles. Thus, *Fig. 6*, the Arch AB of the lesser Circle, as well as the Arch DE of the greater Circle, is a fourth Part of its respective Circle, and it is evident, that the Length of the Legs after no more, as to the Measure of an Angle.

Angle be less than a Right, it is particularly call'd an *acute* Angle; if greater, an *Obtuse*. Thus, *Fig. 4*, ACB and BCD are both right Angles; DCE an *Acute*, ECA an *Obtuse*.

Def. 18.

Contiguous Angles, what.

Two Angles, which have one and the same common Leg, are call'd (\*) *contiguous* Angles; As ACB and BCD, *Fig. 4*, for the Leg BC is common to both.

Def. 19.

Vertical Angles, what.

Two Angles, which touch one another only at their *Vertices* or *Heads*, are call'd *vertical* Angles. Thus, *Fig. 4*, ABC and EBD are vertical Angles; likewise CBD and ABE.

Def. 20.

A perpendicular Line, what.

The Legs of a right Angle are said to be (†) *perpendicular* one to the other. Thus, *Fig. 4*, BC is perpendicular to AC or AB. And whenever the Legs are perpendicular, the Angle contained between them is a right Angle.

Def. 21.

Rectilinear Figures, what.

*Rectilinear* Figures are those, which are comprehended by right Lines. As such are all the Figures spoken of in *Chap. 1*, and *2*, of this Treatise, except the Circle.

(\*) They are stil'd in *Latin*, *Anguli deinceps*.

(†) By a *Perpendicular* is denoted a Line, which inclines no more to one Side than another, but stands equally Upright, and so has the same Position, as a perpendicular (i. e. hanging) Thread, with a Plummer at its End, which rests in, if left to it self. It is also call'd a *normal* Line, from the *Norma* or Square-rule, whose Sides are perpendicular one to the other.



The Lines, whereby Figures are comprehended, are call'd their *Sides*. And bilinear Figures have, either three, or more Sides.

Def. 22.  
The Sides of Figures, what.

Among trilateral (*i. e.* three-sided) Figures, call'd in one Word *Triangles*, an *Equilateral Triangle* is that, which has its three Sides equal one to the other, Fig. 7.

Def. 23.  
An equilateral Triangle, what.

A Triangle, which has only two Sides equal, is call'd an *Isoceles*; as Fig. 8.

Def. 24.  
An Isoceles, what.

A Triangle, which has none of its Sides equal, is call'd a *Scalenum*; as Fig. 9.

Def. 25.  
A Scalenum, what.

A Triangle, which has ( $\parallel$ ) one right Angle, is call'd a *rectangular Triangle*. The Side opposite to the right Angle is call'd the *Hypotenuse* or *Subtense*. As, Fig. 10, A is the right Angle, and the Hypotenuse.

Def. 26.  
A rectangular Triangle, and Hypotenuse, what.

A Triangle, which has an obtuse Angle is thence call'd an (\*) *Amblygonium*; as Fig. 9, where the Angle A is obtuse.

Def. 27.  
An Amblygonium, what.

A Triangle, which has all three Angles acute, is call'd an (\*) *Oxygonium*; as Fig. 7, and 8.

Def. 28.  
An Oxygonium, what.

That no Triangle can have more than one right Angle prov'd from Theorem 4.  
(\*) These Words denote in the Greek Tongue respectively, that which has an *obtuse* or *acute* Angle. And it is noted, that as a Triangle can have no more than one right Angle, so can it have no more than one Obtuse Angle, for the like Reason to be learnt from Theorem 4.

Among

**Def. 29.** Among quadrilateral (*i. e.* four-sided) Figures, a *Square* is that, which has equal Sides, and four right Angles. *Fig. 11.*

**Def. 30.** An ( $\dagger$ ) *Oblong* is that, which has right Angles, but only the two opposite Sides equal; as *Fig. 12.*

**Def. 31.** A *Rhombus* has four equal Sides, four oblique Angles; as *Fig. 13.*

**Def. 32.** A *Rhomboid* has four Sides, and oblique Angles; whereof only the opposite, both Sides and Angles are equal; as *Fig. 14.*

**Def. 33.** Any other quadrilateral Figure, is called a *Trapezium*; as *Fig. 15.*

**Def. 34.** Lines, which are all along equally distant one from the other, are called *Parallels*. Thus, *Fig. 16*, the right Lines AB and CD are *Parallels*; as are *Fig. 17*, the curve Lines ABC and DEF.

**Def. 35.** A *Parallelogram* is any quadrilateral Figure, whose opposite Sides are parallel, and consequently ( $\parallel$ ) equal. Such are all quadrilateral Figures, but a *Trapezium*.

( $\dagger$ ) It is call'd by common Workmen *Long-square* by Mathematicians frequently a *Rectangle* from its right Angles. The other quadrilateral Figure, tho' also all right Angles, being specified by the Name *Quadratum* or *Square*.

( $\parallel$ ) That the opposite Sides of a *Parallelogram* are equal, is wont indeed to be made a Proposition, and to be demonstrated.

Figures, which have more than four Sides, are in general call'd *Polygons*; and distinguished one from another by Letters denoting (in the Greek Language) the respective Number of Angles, and consequently of Sides. Thus, a *Pentagon*, *Hexagon*, *Heptagon*, *Octagon*, &c. denotes respectively a Figure of Five, Six, Seven, Eight, &c. Angles or Sides; as are represented, *Fig. 18, 19, and 21.*

*Def. 36.*

A *Polygon*, *Pentagon*, *Hexagon*, &c. *what?*

Any *Polygon*, that has all its Sides, consequently all its Angles, equal to another, is call'd a *regular Polygon*. Such are the *Pentagon*, *Hexagon*, &c. describ'd, *Fig. 18, 19, 20, and 21.*

*Def. 37.*

A *regular Polygon*, *what?*

As it is obvious, that a *Figure*, considered by it self, is then said to be *equilateral*, when all its own Sides are equal; *equiangular*, when all its own Angles are equal; so it is to be observ'd, that two or more *Figures*, consider'd in respect of one of the other, are then said to be *equilateral*, when the several Sides in

*Def. 38.*

*Equilateral, and equiangular Figures, what?*

proceed. But I have chosen rather to insert it into the Definition of a *Parallelogram*, as being really included therein. For since to be *parallel*, is no other than to be *equidistant*, it follows, that to say, the two opposite Sides of a *Parallelogram* are *Parallel*, is the same in Effect as to say, that the other two opposite Sides are *equal*; it being the Equality of these, that makes the other two *equidistant*, i. e. *parallel*.

one,

one, are respectively equal to the several Sides in the other ; and likewise are said to be *equiangular*, when the several Angles in one, are respectively equal to the several Angles in the other ; though, each Figure, consider'd in it self, is neither equilateral nor equiangular. Thus, *Fig. 22*, and *23*, the Triangles *ABC* and *abc*, are equilateral and equiangular Figures ; because the  $AB=ab$ ,  $BC=bc$ ,  $CA=ca$ , and the Angle  $A=a$ ,  $B=b$ ,  $C=c$ .

**Def. 39.**

Similar  
Figures,  
what.

*Similar* (or *like*) *Figures* are those which have their Angles mutually equal, and also the Sides about the equal Angles ( $\dagger$ ) proportional. Thus the Triangles *ABC* and *abc*, (*Fig. 22* and *25*.) are similar Figures. For the Angle  $A=a$ ,  $B=b$ ,  $C=c$  ; and also Side  $AB : BC :: ab : bc$  ; and  $BC : CA :: bc : ca$  ; and  $CA : AC :: ca : ab$ . And

(\*) It is to be noted, that although equilateral Triangles are equiangular by Theorem 5 ; yet equiangular Triangles are not always equilateral, (as *Fig. 24*, and *25*.) but the Sides about the equal Angles, are always proportional by Theorem 9.

(†) It is suppos'd, that the young Gentleman proceeds in his mathematical Studies regularly ; and therefore has gone through the *young Gentleman's Arithmetick*, and understands what *Ratio* or *Proportion* is, and the several Sorts of it, viz. *Altern*, *Inverse*, &c. *Proportion* ; and what the *Antecedent* and *Consequent* of a *Proportion* are, they being explain'd in the Chapter concerning *Proportion* in the said *Arithmetical Treatise*.



be noted, that all Squares are similar Figures in respect one of the other. Every Square having by *Def. 29*, all four Sides equal, it follows, that if one Side of a greater Square, be the double of any one Side of a less Square, the other Sides of the greater Square, also be the Doubles of all the other Sides of the less Square. And so the Squares will be proportional. And having also both right Angles, they will be similar.

proportional Quantities, the several Antecedents are said to be *homologous* (or alike in their Ratio or Proportion) to the other, and likewise the several Consequents. Thus, if  $A : B :: C : D$ ; then A and C are homologous Quantities, and likewise B and D. And in similar Figures, the Sides that are opposite to the equal Angles, are call'd *homologous Sides*. Thus *Fig. 24*, and *25*, and *ab* are homologous Sides, and *bc*, &c.

To multiply a Line into (or by) a Line, by drawing one Line perpendicular to the other to describe a Rectangle. Thus, *Fig. 11*, and *12*, both

*Def. 40.*

Homologous Quantities, and Sides, what.

*Def. 41.*

To multiply a Line by a Line, what.

hence the Latin Word *duco* (to draw) is used by Latins of *Arithmetick*, for the same as *multiplico*, to

the

the square and oblong Rectangles *AB* are conceiv'd to be produced or multiplied by the Line *AB*, by the *BD*, *i. e.* by drawing *AB* perpendicularly along *BD*. Hence it is said, the Area or Content of a Rectangle *ABCD*, is equal to its two contiguous Sides *AB* and *BD* (or *AB* and *AC* and *CD*, or *CD* and *BD*) multiplied together. Namely, if *AB* be 3 Feet (or Yards or Inches, &c.) and *BD* four, then the Content of *AB* will be  $3 \times 4 = 12$  Feet square. This is illustrated, *Fig. 26*, where, supposing Line *AB* to be drawn from the Point *a* (*viz.* one Foot) upon Line *BD*, it is evident, that by such Motion it will describe the Rectangle *ABeh*, whose Content will be three square Feet. The Line *AB* being mov'd from *e* to *f*, will in like manner describe Rectangle *efih*, containing three square Feet more. And after the same manner will there be describ'd the two Rectangles *fgki*, and *gDCK*, by the Motion of *AB* from *f* to *g*, and from *g* again to *D*; each of these Rectangles evidently containing likewise three square Feet. Whence it evidently follows, that the whole or great Rectangle *ABCD*, contains (four Times three, *i. e.*) twelve square Feet. And this is the most

to be taken Notice of, and clearly  
 rated, because on the Certainty or  
 of this Method of Measuring (or  
 the Content of) Rectangles, do's  
 and the true Measuring of other su-  
 cial Figures; viz. Triangles, Trape-  
 zoids, and Polygons, as will be shewn  
 by the proper Theorems of Chap. 1.

The *Altitude* or *Height* of any Figure, *Def. 42.*

A Perpendicular let down from its  
 Vertex or Top, to its Basis or Bottom.

*The Alti-  
 tude or  
 Height of  
 a Figure,  
 what.*

Thus, the Line AP is the Altitude of  
 Fig. 27, and 28, and 29; where it is to  
 be noted, that the Basis is to be produced  
 so that the Perpendicular will not fall  
 in the Figure, as Fig. 28. Also it  
 is to be noted, that the contiguous Sides  
 of Rectangles (whether Squares or Ob-  
 long) being always perpendicular, there-  
 fore their Altitude is given with them,  
 being always equal to either of the  
 side Sides, which lie between the  
 top and bottom Sides. Thus, AB or CD is  
 the Altitude of the Square, Fig. 11, and  
 of the Oblong, Fig. 12. Lastly, It is to  
 be noted, that the Altitude of any Ob-  
 ject (viz. Tree, Tower, &c.) is like-  
 measur'd by a Perpendicular, from  
 its Top to its Bottom.

AXIOMS.

## A X I O M S.

- Ax.* 1. The Whole is greater than any of it.
- Ax.* 2. The Whole is equal to all its Parts taken together.
- Ax.* 3. Quantities, which are equal to same Quantity, are equal between themselves. If  $A=B=C$ , then  $A=C$ .
- Ax.* 4. Quantities, which are (\*) proportional to the same Quantities, are proportional between themselves. If  $A : B :: C : D :: E : F$ , then  $A : B :: E : F$ .
- Ax.* 5. Figures, which are (\*) similar to same Figure, are similar between themselves.
- Ax.* 6. The same Quantities, have the same Proportion to equal Quantities; and equal Quantities, have the same Proportion to the same Quantities. If  $C=D$ , then  $A : C :: A : D$ . Or, if  $C=E$ ,  $D=F$ , and  $A : B :: C : D$ , then  $A : B :: E : F$ .
- Ax.* 7. If equal Quantities be added to equal Quantities, the Wholes will be equal.

---

(\*\*) The Terms, *Proportional* and *Similar*, being understood as well as the Term *Equal*; what is here said of proportional and similar Quantities, is as Self-evident as what is said of equal Quantities; and therefore as justly be esteem'd to many *Axioms*.



If  $A=B$ , then  $A+C=B+C$ . Or, if  $A=B$ , and  $C=D$ , then  $A+C=B+D$ .

If equal Quantities be subtracted from equal Quantities, the Remainders will be equal. *Ax. 8.*

If  $A=B$ , then  $A-C=B-C$ .

If  $A=B$ , and  $C=D$ , then  $A-C=B-D$ .

If Quantities mutually proportional be added together, the Wholes will be proportional. *Ax. 9.*

If  $A:C::B:D$ , and  $A:E::B:F$ , then  $A:C+E::B:D+F$ . Or

If  $A:B::C:D::E:F$ , then  $A:B::C+E:D+F$ ; and also  $A:B::C+E:B+D+F$ . *&c.* And the

same holds Good in respect of *Subtraction* as well as *Addition*.

The Equi-multiples (*viz.* Doubles, Triples, *&c.*) of the same or equal Quantities, are themselves equal. As

If  $A=B$ , then  $2A=2B$ ,  $3A=3B$ ,  $4A=4B$ , *&c.*

The Equi-multiples (*viz.* Doubles, Triples, *&c.*) of any two Quantities, are proportional to the Quantities themselves. Namely,  $A:B::2A:2B::3A:3B$  *&c.*

The like Parts (*viz.* Halves, Thirds, Fourths, *&c.*) of the same or equal Quantities, are themselves equal. Namely, as  $A=A$ ,  $\frac{1}{2}A=\frac{1}{2}A$ ,  $\frac{1}{3}A=\frac{1}{3}A$ , *&c.* And if  $A=B$ ,  $\frac{1}{2}A=\frac{1}{2}B$ ,  $\frac{1}{3}A=\frac{1}{3}B$ , *&c.*

The

**Ax. 13.** The like Parts (*viz.* Halves, Thirds, &c.) of any two Quantities, are proportional to the Quantities themselves. Namely,  $A : B :: \frac{1}{2}A : \frac{1}{2}B :: \frac{1}{3}A : \frac{1}{3}B$  &c.

**Ax. 14.** If one Quantity be the Double, Triple, &c.) of another Quantity, what is taken away from the greater Quantity, be likewise the Double, Triple, &c.) of what is taken away from the lesser Quantity; then the Remainder of the greater Quantity, will be the Double (or Triple, &c.) of the Remainder of the lesser Quantity.

**Ax. 15.** Quantities, which apply'd to, or upon one the other mutually agree, so as all the Parts of the one, do equally answer to all the Parts of the other, are equal between themselves.

**Ax. 16.** All right Angles are equal one to another.

**Ax. 17.** The Rays of equal Circles, are equal.

**Ax. 18.** Equal Arches of equal Circles, are equal; and consequently, Angles subtend'd by such equal Arches, are equal.

**Ax. 19.** Perpendiculars to one and the same right Line, are ( $\dagger$ ) Parallels one to another.

( $\dagger$ ) For to any one that has a clear Notion of a Perpendicular, it is Self-evident, that the said Perpendicular must all along keep at the same Distance one from the other, as are the Points, upon which they fall, in the Line to which they are perpendicular.

# THEOREM I.

The Sum of any two contiguous Angles ABC, and ABD, made by a right Line CD, falling upon a right Line AB, is always equal to two right Angles. *Fig. 30.*

*Demonstration.* For it is evident, that the Quantity of the two Angles ABC and ABD put together, make up the Measure of the Semi-circumference CAD, so are equal to 180 Degrees, which is the Measure of two right Angles, by

*Corollary 1.* If one of the two contiguous Angles be right, the other will be right also. If one be obtuse, the other ABD will be obtuse; contrariwise, if one ABD be obtuse, the other ABC will be acute. For

the Quantity of both Angles together is 180 Degrees. And therefore, if one Angle be 90 Degrees, the other must be 90 also: If one be less than 90, the other must be so much more than 90; and contrariwise, if one be more than 90, the other must be so much less.

*Cor. 2.* Contiguous Angles, if they be both Right. For (by *Theorem 1.*) they are together equal to 180 Degrees. And therefore, being both equal

one to the other, each must be 90 Degrees, the Measure of a right Angle.

*Cor. 3.* The Sum of all the Angles that can be made by right Lines, standing upon (i. e. not crossing) one and the same Point B, of a right Line CD is equal to two right Angles. For it is evident, that the Sum of all the Angles CBA, ABE, EBF, GBH, and HBD is just equal to (the Semi circumference CAEFGHD, or) 180 Degrees.

*Cor. 4.* The Sum of all the Angles made by two or more right Lines, crossing each the other in one and the same Point B, is equal to four right Angles. For they evidently take up the whole Circumference of their respective Circles. As do the four Angles, ABC, CBD, DBE, and EBA, in *Fig. 33*, made by the two Lines AD and CE, crossing each other in B; and the eight Angles made by the four Lines AF, CG, DH, and EI, crossing in the Point B, *Fig. 34*.

### U S E.

The first Theorem, and its Corollaries are not only of great, and as it were fundamental Use in demonstrating Propositions, but also of great Use in Practice. For Instance, by Corollary known, what regular Surfaces (wh



Common Stone, or Marble, or Wood, may be join'd together in Pavements; namely, those only, whose Angles, taking a certain Number of them together, are just equal to four Right Angles. Such are equilateral Triangles, Squares, and regular Hexagons. For the Angle of an equilateral Triangle, being by *Theorem 4.* and *Theorem 5. Corol. 3.* always equal to one Third of two Right Angles, or to 60 Degrees; and the Angle of a Square, being by *Def. 29.* always a right Angle; and the Angle of a regular Hexagon, being (by *Corol. 4. Theorem 5.*) always equal to one sixth Part of eight Right Angles, i. e. to 120 Degrees; therefore, six equilateral Triangles, or four Squares, or three regular Hexagons, will join together in one Point. For  $60 \times 6 = 360$ ,  $90 \times 4 = 360$ , and  $120 \times 3 = 360$ , and the Quantity of all the Angles, that can be about one Point, is 360 Degrees, by *Theorem 4.* of this *Theorem 1.* This is illustrated, *Fig. 35.* as to Squares, and *Fig. 36.* as to Triangles, and *Fig. 37.* as to Hexagons; in each of which Figures, *P* denotes the Point, wherein the several Squares, or Triangles, or Hexagons

## THEOREM II.

Fig. 38. Vertical Angles CEB and AED, equal one to the other,

*Dem.* For by *Theorem 1*, the Ang  
 $AEC + CEB = 2 \text{ Right.} = AEC + AED$   
 Therefore, by *Axiom 3*,  $AEC + CEB = AEC + AED$ . Therefore, by *Axiom 1*,  
 $CEB - AEC = AED - AEC$ , i. e.  $CEB = AED$ .

## USE.

1.

*The Use of this Theorem 2d, illustrated, in taking the Sun's Height, and the Angle of Distance between two Objects.*

This second Theorem is of great Use not only as it serves to demonstrate other Propositions, but also as on it depends the Demonstration of the Method of taking the Sun's Height by the Quadrant, and also of taking the Angle of Distance between two Objects, by the Theorem of Circumferentor, and other like Instruments.

2.

*The Method of taking the Sun's Height by the Quadrant, demonstrated.*

Fig. 39.

That the Sun's Height, or Distance above the Horizon, is truly taken by the Quadrant, is illustrated, Fig. 39: the Ray of the Sun, coming to the Eye, through the Sights of the Quadrant, makes a right Line AB, call'd the Line of Vision. And the String of the Quadrant, to which the Plummets is fastened, suppos'd to be continu'd to the Zenith, when it hangs true or exactly perpendicular.

er, makes another right Line DE.  
 These two Lines crossing one the other at  
 the Center of the Quadrant, make  
 two vertical Angles ACD and BCE.  
 These two Angles, being by this Theor.  
 equal one to another, it thence fol-  
 lows, that the Degrees between the Zen-  
 ith and the Center of the Sun, are just  
 many as the Degrees between the  
 String and that Edge of the Quadrant on  
 which the Sights are. And therefore,  
 supposing the Degrees between the String  
 and the said Edge of the Quadrant to be  
 the Distance of the Sun (*i. e.* of the  
 Center of the Sun) from the Zenith will  
 be 60 Degrees likewise. Now the Dis-  
 tance of the Zenith from the Horizon  
 is always 90 Degrees, which is the  
 Number of Degrees in a Quadrant; it  
 follows, that as the Distance of the Sun  
 from the Horizon, (*i. e.* in one Word,  
 Height,) must be always equal to the  
 Distance, arising from the Subtraction of  
 the Distance between the Sun and the  
 Zenith out of 90; so it must be always  
 equal to the Number of Degrees, be-  
 tween the String and the other Edge of  
 the Quadrant, which has not the Sights  
 on it; because, the said Number of De-  
 grees on the Limb of the Quadrant is  
 the Residue, arising from the Sub-  
 traction of the Distance between the Sun

and the Zenith from 90. According supposing the Sun's Distance from Zenith to be 30 Degrees, its Height be 60 Degrees; and just so many degrees will be intercepted between String of the Quadrant, and that Edge it, which has not the Sights; as may be seen, *Fig. 39.*

3.  
*The Method of taking the Angle of Distance between two Objects, demonstrated.*  
*Fig. 40.*

That the Distance between two Objects is truly taken by the Theodolite (or the like Instrument,) is illustrated *Fig. 40.* For the Rays of Vision coming through the Sights of the Theodolite make two right Lines AB and DE crossing one another at C, the Center of Theodolite; and so making these vertical Angles ACE and DCB, which the Angle ACE, being the Angle of Distance between the two Objects, and being by this Theorem ad, equal to the Angle DCB, it follows, that the Measure of the Angle DCB, viz. Degrees, is also the true Measure of the Angle of Distance ACE, as it is also shewn to be upon the Theodolite, *40.*

### THEOREM III.

A right Line EF falling (either perpendicularly, as *Fig. 41.*, or obliquely, as *Fig. 42.*) upon two right Lines



If one to the other  $AB$  and  $CD$ , will  
 be the extern Angle  $ABE$ , equal to  
 intern and opposite  $CDE$ .

*Demon.* If the right Line  $EF$  falls per-  
 pendicularly on the Parallels  $AB$  and  $CD$ ,  
*Fig. 41,*) then by *Def. 20*, the An-  
 gles  $ABE$  and  $CDE$ , are both Right, and  
 consequently equal by *Axiom 16*.

*Fig. 41.*

If the Line  $EF$  falls obliquely on the  
 Parallels  $AB$  and  $CD$ , (as *Fig. 42,*) then  
 the prick'd Lines  $BG$  and  $DH$  be both  
 perpendicular to the Line  $EF$ ; or,  
 which comes to the same, be two Par-  
 allels, on which the Line  $EF$  falls per-  
 pendicularly, (as it do's on the Parallels  
 $AB$  and  $CD$ , *Fig. 41,*) hereupon the  
 Angles  $EBG$  and  $EDH$ , will be by *Def.*  
 both right, and therefore, by *Axiom*  
 equal. Now, supposing the perpen-  
 dicular Parallels  $BG$  and  $DH$ , may be  
 turn'd about upon their respective Points  
 $B$  and  $D$ , (as they are actually made to  
 turn'd about in the common Parallel  
 $EF$ , whereby, what is here said, may  
 be evidently represented to the very  
 eye.) It will follow, that as much as the  
 Angle  $EBG$  is lessen'd by turning  
 the perpendicular Parallel  $BG$ , till it  
 comes to the oblique Parallel  $AB$ , so

*Fig. 42.*

How to draw a Perpendicular, See Chap. 2, Pro-  
 6.

much

much must the other right Angle be lessen'd by the equal turning of other perpendicular Parallel DH, (being suppos'd to keep all along parallel BG,) till it comes to the other oblique Parallel CD; and consequently, the Angles EBA and EDC, being the Remainers of two right Angles EBG and EDH, equally lessen'd, must be equal one to another by *Axiom 8.*

*Cor. 1.* A right Line EF falling to two Parallels AB and CD, makes two intern Angles ABF and CDE equal to two Right. This is evident, if EF falls perpendicularly on AB and CD. *Fig. 41.* For then the Angles ABF and CDE are each right Angles by *Def.* and so equal by *Axiom 16.* In like manner, *Fig. 42,* the Lines BG and DH being Perpendiculars to EF, the Angles GBF and HDE are two Right by *Def.* so. Wherefore, the Lines BG and DH being suppos'd (as afore, *Corol. 1.*) to turn round upon their Points B and D till they come to be in the same oblique Position with AB and CD, and to be all the while Parallel one to the other, it is evident, that as much as the Angle CDB, becomes less than the right Angle HDB, so much the Angle ABF, becomes greater than the right Angle GBF; consequently, the two oblique intern

oppo

the Angles  $ABF$  and  $CDB$ , are to-  
 er equal to two Right, in what De-  
 soever of Obliquity the Line  $EF$   
 upon the two Parallels  $AB$  and  $CD$ .  
 fore-going Demonstration of Corol.  
 founded on the more natural or im-  
 ate Reason of the Things proved.  
 said Corollary may be otherwise de-  
 strated universally (i.e. whether the  
 $EF$  falls perpendicularly or obli-  
 y on the Parallels  $AB$  and  $CD$ .) by  
 and the same Manner of Proof, thus:  
*Theorem 3*, the Angle  $EBA = EDG$ ,  
 by *Theorem 1*,  $EBA + ABF = 2 \text{ Right}$ ,  
 before, by *Axiom 7*,  $EDG + ABF = 2$

*Corol. 2* A right Line  $EF$  falling upon *Fig. 43,*  
 Parallels  $AB$  and  $CD$ , makes the al- & 44.  
 Angles  $AGF$  and  $EHD$  equal one to  
 other. If the Line  $EF$  falls perpendi-  
 cularly upon the two Parallels  $AB$  and  
 as *Fig. 43*, then it is evident, that  
*Axiom 16*, the altern Angles are e-  
 qual, because they are both Right by  
*Def. 20*. And from hence it may be  
 shew'd, that the altern Angles will be  
 equal likewise, though oblique, (as *Fig.*  
*44*) forasmuch as the said oblique al-  
 tern Angles  $AGF$  and  $EHD$ , may be con-  
 sider'd to be made by the equal turning  
 of the perpendicular Parallels  $KL$  and  
 from their perpendicular Position,  
 to

to the oblique Position of the Parallel Lines  $AB$  and  $CD$ . But this *Corol. 2*, is otherwise prov'd universally, thus: By *Theorem 2*, the Angle  $AGF = EOB$ , and by *Theorem 3*,  $EOB = EHD$ , therefore, *Axiom 3*,  $AGF = EHD$ .

Fig. 43,  
& 44.

*Corol. 3*. If a right Line  $EF$  falling on two right Lines  $AB$  and  $CD$ , make the extern Angle  $AGE$ , equal to the intern opposite  $CHE$ , or the altern Ang  $AGF$  and  $EHD$  equal one to the other, or the two intern Angles  $AGF$  and  $CHE$  equal to two Right, then the Lines  $AB$  and  $CD$  are parallel one to the other. (This *Corollary* is no other than the Converse of this *Theorem 3d*, and the two fore-going *Corollaries*, and all the Truth of it sufficiently appears from thence, yet, it may be more evidently prov'd thus:) If in the three fore-mention'd Cases, the two Angles composed together be both right, as *Fig. 43*. (If the extern and intern Angles be both right, or the two Alterns be both right, or the two Interns be both right,) then by *Def. 20*, the Lines  $AB$  and  $CD$  are both perpendicular to  $EF$ , and therefore by *Axiom 19*, parallel one to the other. Wherefore, they will also keep parallel if they be equally turn'd from a Perpendicular to any oblique Position; which is the same as to say, that if (as in



the extern  $\angle ACE$  and intern  $\angle CHE$  equal oblique Angles, or if the two  $\angle ACF$  and  $\angle EHD$  be equal oblique Angles, or if the two oblique Intern  $\angle ACF$  and  $\angle CHE$  are together equal to two right Angles, then the Lines  $AB$  and  $CD$  are parallel (\*).

U S E.

This third Theorem (with its Corollary) is of great Use; as in demonstrating other Propositions, so also in demonstrating more immediately some Methods in Practice. For Instance: By this Theorem may be demonstrated, in taking the Height of an Object by the Quadrant, the Number of Degrees between that Edge of the Quadrant, on which the Sights are, and the Edge, do truly contain the Measure of the Angle  $CBA$ , made by  $CA$  the Perpendicular (or Altitude) of the Object, and  $BC$  the Line of Vision. For, when the String of the Quadrant hangs true, and perpendicular, it will be parallel to  $CA$  the Perpendicular of the Object. Therefore, the Angle  $baa$ , made by the Line of Vision and the String of the

*The Method of taking the Angle of Altitude of an Object by the Quadrant, partly demonstrated, viz. as to the Angle made by the Ray of Vision, and Perpendicular of the Object.*  
Fig. 45.

See another Proof of this Corollary, in the Notes on the following Theorem the 4th.

Quadrant,

Quadrant, will be the extern Angle the Angle BCA, and consequently, will be equal one to the other by third Theorem. How from hence other Angle ABC is known, will be taught in Corol. 2, of the following Theorem, and will be illustrated in Use of the said Theorem.

### THEOREM IV.

Fig. 46.

Of any Triangle ABC, one Side being produc'd, the extern Angle A is equal to the two intern and opp A and B. And consequently, all three intern Angles, viz. A and B, ACB, are together equal to two Right.

*Demon.* Upon C, let the Line CD drawn (+) parallel to BA. There the Angle  $A = ACE$ , by Corol. 2, Theorem 3. And the Angle  $B = ECD$ , by Theorem 3. Wherefore, by Axiom 7,  $ACE + ECD = ACD$ , (by Ax. 2 :) Was the first Particular to be demonstrated. To proceed then, the Angle  $BCA + ACE + ECD = BCA + A + B$ , by Axiom 7. But the Angles  $BCA + ACE + ECD = 2$  Right by Corol. 3, Theorem 3. Therefore, the Angles  $BCA + A + B = 2$  Right, by Ax. 3.

(+) How to draw a Parallel, See Chap. 2, Problem 1.

Prop. 1. In any Triangle, if one Angle be right, the other two together make up the Quantity of another Right.

Prop. 2. The Quantity of any two Angles in a Triangle being known, the Quantity of the third Angle is also known by; namely, it is the Complement of the other two Angles to two Right, 180 Degrees.

Prop. 3. If two Angles in one Triangle be equal to two Angles in another Triangle; then the other remaining Angles in each Triangle, will also be equal to the other (Q.E.D.).

What is above set down under Theorem 3, as its Corollary, may be otherwise prov'd from this Theorem in this manner. If, when the Altern Angles EHD be equal, it be denied, that the Lines AB and CD are parallel; then, suppose the said Lines to meet in P, and the Figure HPG to be a Triangle; to AGF will be an extern Angle; and therefore, by Prop. 4,  $AGF = GHP$  and  $HPG$ , and therefore bigger than  $GHP$  alone, or which is the same bigger than EHD, which it was at first suppos'd to be equal. But it is inconsistent, that the angle EHD, should be both equal and lesser than the Extern AGF, it follows, that it is equal, (as was first suppos'd,) therefore the Lines AB and CD will never meet, but are parallel. And it follows, that if the Extern EGB, and Intern EHD be equal, AB and CD are Parallels: For  $EGB = AGF$  (by Prop. 2,) the Altern to EHD. And so lastly, if the Extern AGF and CHE, be equal to two Right: For  $CHE = a$  Right. Wherefore,  $DNE = AGF$ , its Altern

Fig. 47.

U S E.

## USE.

Besides the great Use of this Theorem (and its Corollaries) in demonstrating other Propositions, on it immediately depends several Particulars of great Use in Practice, some of which I must be content here to take notice of.

1.  
The Method of taking the Height of an Object by the Quadrant, demonstrated.

Hereby, then is demonstrated, the Angle of the Height of any Object truly taken by the Quadrant. For 45,) the horizontal Line BA, and Perpendicular of the Object CA, and Ray of Vision BC, do make together Triangle ABC. In which the Line BA being perpendicular to the Line CA, it follows, by *Defin. 20*, that the Angle BAC is a right Angle. And therefore by *Corol. 1*, of this Theorem, the two Angles do together make up a right Angle. But it has been afore (in the Use of *Theorem 3*,) shewn, that the Angle bca on the Quadrant, is equal to the Angle BCA in the Triangle ABC. Therefore, it follows, by *Corol. 2*, of this Theorem, that the third Angle ABC in the Triangle ABC, is equal to the Complement of the Angles BAC and BCA, that is, to the Angle ace of the Quadrant. For, supposing the



( $\angle bca$ ) to be 60 Degrees, the An-  
 $\angle ABC$  ( $\angle acb$ ) will be found by the  
 drant to be 30 Degrees, which is the  
 plement of the  $\angle BAC$  90, and  
 60, to two Right, viz.  $90 + 60 + 30$   
 180. *Q. ad. B. ad. A. xii. h. id. w.*  
 further, by Help of this Theorem  
 2. principally it is demonstrated, that all  
 Angles of any right lin'd Figure, do  
 ther make up the Quantity of twice  
 many right Angles, abating four, as  
 Figure has Sides. For any such Fi-  
 gure may be divided into so many Tri-  
 angles, as it has Sides. Wherefore, since  
 angles of each Triangle, will by this  
 Theorem be equal to two Right; the  
 Sum of all the Angles in all the Tri-  
 angles, into which the Figure is divided,  
 together be equal to twice as many  
 Angles, as the Figure has Sides.  
 Now the Angles about P, the inward  
 angles of each Figure, wherein all the  
 Angles concur, are (by Corol. 4. Theor.)  
 equal to four Right. Therefore, if  
 the Sum of the Angles of all the  
 triangles, into which any Figure is re-  
 duced, you take away the Angles about  
 Point P, the Rest of the Angles,  
 will make up the Angles of the Figure,  
 and will make up together the Sum of twice  
 as many Right, abating four, as the Fi-  
 gure has Sides. Thus, Fig. 49. the Fig. 49  
 P Triangle

2.  
 The Me-  
 thod of  
 finding  
 the Sum  
 of all the  
 Angles in  
 any Poly-  
 gon, de-  
 monstra-  
 ted.

Triangle ABC is resolv'd into three angles PAB, PBC, and PCA; the angles of each of which by this Theorem are equal to two Right; and therefore are all together equal to six Right. From which six Angles, deduct the Quantity of the Angles at P, equal (by Theorem 1. 4.) to four Right, and there will remain the Quantity of two right Angles, the Sum of the three Angles of the angle ABC, agreeably to this Theorem.

3. *Fig. 50.* So the Square ABCD being resolv'd into four Triangles PAB, PBC, PCD, and PDA, the Angles in each Triangle will be equal to two Right; and for the Sum of the Angles in all the several angles will be equal to eight Right. From which deduct the Quantity of four Right, which is the Sum of the Angles about P, and there will remain the Quantity of four right Angles, for the Sum of the four Angles of the Square ABCD, agreeably to *Defin. 29.*

4. *Fig. 51.* Once more, the Hexagon, being resolv'd into six Triangles, the angles of each will be together equal to two Right; and for the Sum of all the angles in the six Triangles, will together be equal to twelve Right. From which deduct the Quantity of four Right, for the Sum of the Angles at P, and there will remain the Quantity of eight Right,

Sum of the six proper Angles of the polygon.

# THEOREM V.

Two Triangles  $ABC$  and  $abc$ , having several Sides mutually equal, *viz.*  $Fig. 52$   
 $AB=ab$ ,  $BC=bc$ , and  $CA=ca$ , will have 53.

the Angles, contain'd between the mutually equal Sides, equal one to the other, the Angle  $A=a$ ,  $B=b$ ,  $C=c$ , and Triangles themselves will be mutually equal. In short, thus: Triangles mutually equilateral, are also mutually equiangular, and equal one to the other.

*Demon.* For the Side  $AB$  being put on  $ab$ , and equal thereto,  $BC$  will fall on  $bc$ , and  $CA$  upon  $ca$ , by Reason of their respective mutual Equality. And several Sides of the Triangle  $ABC$ , falling exactly upon the corresponding Sides of the Triangle  $abc$ , it evidently follows, that any two contiguous Sides of one Triangle, must have the same Inclination one to another, as have two contiguous Sides of the other Triangle upon which they fall, (*viz.*  $AB$  have the same Inclination to  $BC$ , as  $ab$  to  $bc$ , and so of the rest;) and therefore by *Defin.* 14, the Angle contain'd between each two contiguous Sides of the Triangle, must be equal to the Angle

gle contain'd between the respond-  
Sides of the other Triangle ; and so  
two Triangles be mutually equiangul-  
as well as mutually equilateral. And  
they are also by *Axiom 15*, mutually  
equal, forasmuch as the Sides of one,  
ling exactly upon the correspondent Sides  
of the other, the Sides of each must  
clude or take up equal Space.

Fig. 54,  
55.

*Corol. 1.* Two Triangles DEF and  
having an Angle E, equal to an Angle  
and the Legs of the said Angle mutu-  
equal, (*viz.*  $ED = ed$ , and  $EF = ef$ .)  
be equal in all other Respects, *viz.*  
(\*) Basis DF, will be equal to the Basis  
 $df$ , and the Angle  $D = d$ , and  $F = f$ .  
For the Angle E being put upon  $e$ ,  
Leg ED will fall upon  $ed$ , and EF upon  
 $ef$ ; and the said Legs being equal,  
right Lines, the Point D will fall upon  
 $d$ , and the Point F upon  $f$ . Wherefore  
the two Bases DF and  $df$ , being  
right Lines drawn from the Points D  
 $d$  thus lying one upon another, to  
Points F and  $f$  lying also one upon

---

(\*) Any angular Point of a Triangle (as E or  $e$ )  
taken for the Vertex of the Triangle, the Side oppo-  
site the said angular Point, is call'd the *Basis* of the Triangle.  
In an Isosceles, the angular Point, wherein the two  
Sides meet, being usually taken for its Vertex, hence  
unequal Side is more peculiarly call'd the Basis of the  
Isosceles.



must likewise themselves lie one in another, and be equal, according Defn. 3. For, according thereto, they being both right Lines, must both lie exactly even between the said Points, therefore the Points D and d, as also e and f, lying one upon the other, the Lines DF and df drawn exactly even between, and only from and to the said Points, must also co-incide (or fall one in the other) and be equal one to the other. Wherefore, the Triangles EDF and edf being mutually equilateral, it follows from this Theorem 5, that they are also mutually equiangular, and e-

Prop. 2. In an Isosceles Triangle ABC, Fig. 56. Angles ABC and ACB, at the Basis are equal. For, supposing the Line AD to bisect the Angle A, it will follow, that the Triangles ABD and ACD, have the Angles BAD and CAD equal. And they having also the Sides BA and AC equal, and the Side AD common to both; it will follow by Prop. 1, of this Theorem, that the said Angles are mutually equiangular, and consequently the Angle (ABD, or which is the same) ABC = (ACD, or) ACB.

N. B. Hence it follows also, that a Triangle ABC, which has the Angles the Base ABC and ACB mutually equal, hath also the other two Sides AB and AC equal.

*Corol. 3.* Every equilateral Triangle is also equiangular, i. e. has all its three Angles equal one to the other. an equilateral Triangle, is an Isosceles every Way. Wherefore, *Fig. 57.*

*Fig. 57.*

being taken for the Basis, by *Corol. 2.* this Theorem, the Angle  $ABC = ACB$  and CA being taken for the Basis, Angle  $ABC = CAB$ ; and AB being taken for the Basis, the Angle  $CAB = ACB$ . Wherefore,  $ABC = ACB = CAB$ .

*Corol. 4.* The Angles of any regular Polygon, are all equal one to the other. For any regular Polygon, may be divided into so many Isosceles, and mutually equilateral Triangles, as it has Sides, and consequently the said Triangles be (by this Theorem) mutually equiangular. And therefore, the Angles of the Polygon (consisting each of two Angles mutually equal in the said Triangles) be equal one to the other. Thus in the Hexagon, *Fig. 51.* the Triangles PBC, PCD, &c. are each an Isosceles in it self, and also mutually equilateral and so mutually equiangular. For this Theorem 5, the Angle  $PBA = PCD$ .

Corol. 2. of this Theorem)  $PCB =$   
 $D$ , by this Theorem 5, it self. Where-  
 by, by *Axiom* 7,  $PBA + PBC = PCB +$   
 $D$ , i. e.  $ABC = ECD$ .

Corol. 5. A (||) Diagonal  $CB$ , divides  
 Parallelogram  $ABCD$  into two equal  
 Triangles  $ABC$  and  $CDB$ . For the Sides  $AB$   
 $CD$  being equal by *Defn.* 13, and  
 the Sides  $AC$  and  $BD$  and the Side  
 being common, it follows by this  
 Theorem, that the Triangles  $ABC$  and  
 $CDB$  are equal, and consequently that  
 Parallelogram  $ABCD$ , being divided  
 by the Diagonal  $CB$  into the said two  
 Triangles, is by the said Diagonal divid-  
 ed into two equal Parts.

Cor. 6. In every Parallelogram  $ABCD$ ,  
 Complement  $DG$  and  $GB$  of the Pa-  
 rallelograms  $HE$  and  $EF$  about the Dia-  
 gonal  $AC$ , are equal one to the other.  
 by the fore-going Corol. 5, the Tri-  
 angle  $ACD = ACB$ , and the Triangle  
 $AGE = AGE$ , and the Triangle  $GCF =$   
 $GCF$ ; therefore, by *Axiom* 8, the Com-  
 plement  $DG = GB$ .

Fig. 38

Fig. 39

The Word *Diagonal* do's import a right Line drawn  
 from one Angle C of it, to its  
 opposite Angle B.

1.  
In Bisecting Angles or Lines.

2.  
In finding the Quantity of each Angle in an equilateral Triangle.

3.  
In finding the Quantity of each Angle of a regular Polygon.

Besides the great Use of this 5th Theorem and its Corollaries in demonstrating other Propositions, they are all of great Use in Practice. For on this Theorem and its two first Corollaries is founded the Method of Bisecting a given Angle or Line, as also of Drawing a Perpendicular to a given Line. And by Corol. 3, is known the Quantity of each Angle in an equilateral angle. For all the Angles of a Triangle being by Theorem 4, equal to Right, or 180 Degrees; and the Angle of an equilateral Triangle, being by Corol. 3, of this Theorem 5, equal one to another, it hence follows, that each Angle of an equilateral Triangle, is equal one third Part of two right Angles, or to 60 Degrees. And hence is clear what was above taken Notice of in the Use of Theorem 1, viz. that six equilateral Triangles will meet together at a point of Pavement.

In like manner, by Corol. 4, of this Theorem, is discover'd the Quantity of each Angle of any regular Polygon. It has been shewn in the Use of Theorem 4, that all the Angles of any right-angled Figure, are equal to twice as many



ht, as the Figure has Sides, abating  
 r. Accordingly, the Angles of a  
 ragon, make up together the Sum  
 six right Angles, or 540 Degrees.  
 erefore, by *Corol. 4.* of this Theorem,  
 n Angle of a regular Pentagon, is e-  
 l to ( $\frac{3}{2}$ , i. e.) 180 Degrees. In  
 manner, all the Angles of an Hexa-  
 , amounting to the Quantity of eight  
 t Angles, or to 720 Degrees, each  
 le of a regular Hexagon, is equal to  
 (=) 120 Degrees. So the Angles of  
 Heptagon, being all together equal to  
 Right, and of an Octogon to twelve  
 ht, hence each Angle of a regular  
 ragon, will contain ( $\frac{7}{2}$  of ten right  
 les =) 128 Degrees, and  $\frac{1}{2}$  of a De-  
 ; and each Angle of a regular Octo-  
 will contain ( $\frac{3}{2}$  of twelve Right =)  
 Degrees. And so of other regular  
 ygons. And hence is learn'd, what  
 ove (in the Use of Theorem 1,) ta-  
 Notice of, *viz.* that three regular  
 agons will join together in a Point of  
 cement: For  $120 \times 3 = 360$ , which is  
 Quantity of all the Angles that can  
 made or meet in one Point. And  
 e also it may be learn'd, that three  
 lar Pentagons will not fill up a Point  
 round, (for  $108 \times 3 = 324$ .) and four  
 lar Pentagons can't be made to meet  
 Point; for  $108 \times 4 = 432$ . And so  
 of

of other regular Polygons, besides Hexagon.

4.  
In Bisection  
Parallelo-  
grams, &c.

By Cor. 5. of this Theorem, is the Method of Bisection any Parallelogram, namely, by drawing a Diagonal. And on Corol. 6. depends the Method making a Parallelogram, equal to a Triangle, or other rectilinear Figure on a given Line.

## THEOREM VI.

Fig. 60,  
61, 62.

An Angle BDC, at the Center, is Double of an Angle BAC at the Circumference, having both the same Arch for their Basis: Or which comes to the same, an Angle BAC at the Circumference, is Half of an Arch BDC at the Center, both standing upon the Arch BC.

Fig. 60.

*Demon. Case 1.* When the Angles are made, as Fig. 60; Where DC and AC being Rays of the same Circle, the angle DCA is an Isosceles, and consequently by Corol. 2, Theorem 5, the angle  $DCA = DAC$ . Wherefore, by Theorem 7,  $DCA + DAC = 2DAC$ . But by Theorem 4, the Angle  $BDC = DCA + DAC$ ; therefore, by Axiom 3,  $BDC = 2DAC$  and consequently, by Axiom 8,  $DAC = \frac{1}{2}BDC$ .

Fig. 61.

2. When the Angles are made, as

where, the Diameter AE being

there will be (by *Defin. 9.* and

*Theorem 3.* and *Theorem 4.*)

Angle  $BDE = DAB + DBA = 2DAB$ .

*Axiom 7.* Likewise the Angle  $EDC =$

C. Therefore, the whole Angle

$= 2BAC$ , by *Ax. 7.* and so  $BDC$

C, by *Axiom 8.*

3. When the Angles are made, as

62, then (by the same *Definition*,

*Theorems*, and *Corollary*, and *Axiom*

as afore) the Angle  $EDC = DAC +$

$= 2DAC$ . Also the Angle  $EDB$

Part of the Angle  $EDC = DAB$

$BA = 2DAB$ . Wherefore, by *Axiom*

the Angle  $BDC$  (the Residue of the

Angle  $EDC) = 2BAC$ . And therefore,

Angle  $BAC = \frac{1}{2}BDC$ , by *Axiom 8.*

1. Angles  $DAC$  and  $DBC$ , which

in the same Segment  $DABC$ , or

stand upon the same Arch  $DC$ ,

equal. For by this *Theorem*, they

each the Halves of the said Arch

2. Any Angle  $ABC$ , which is

Semicircle  $DABC$ , for which has a

diameter  $AC$  for its Basis, or which

is upon the Semi-circumference  $ABC$

Circle, is a right Angle. For by this

from its Measure is Half the Semi-

circumference which it stands upon, i. e.

90 Degrees, the Measure of a right  
gle.

Fig. 65.

*Corol. 3.* The two opposite Angles  
ABC and ADC of any quadrilatera-  
gure, describ'd in a Circle ABCD  
equal to two Right. For the Arch  
whereon the Angle ADC stands, and  
Arch ADC whereon the other Angle  
stands, make up together the whole  
circumference of the Circle ABCD.  
Half whereof is equal to 180 Deg.  
the Quantity of two right Angles.

### U S E.

Amongst the other Uses of this Theorem and its Corollaries in Practice as well as in demonstrating other Propositions, it is remarkable, that on Corol. 3. is founded the Method of erecting a perpendicular at the End of a Line given, and consequently of trying whether a Square-rule be made true, or no. From Corol. 3, it is learn'd, that a Circle can't be describ'd about a Rhombus, inasmuch as the opposite Angles of that figure are either greater or less than Right,



# THEOREM VII.

Parallelograms BCDA and BCFE, upon the same Basis BC, and within the Parallels AF and BC, are equal one to the other. *Fig. 66.*

*Dem.* By *Defin.* 35,  $AD = BC = EF$ . Therefore, by *Axiom* 7,  $AD + DE = EF$ , i. e.  $AE = DF$ . Likewise  $AB = DC$ , by *Defin.* 35, and the intern Angle  $ABE = DCF$ , by *Theorem* 3. Therefore the Triangle  $ABE = DCF$ , by *Corol.* *Theorem* 5. Therefore, by *Axiom* 8, Triangle  $ABE - DGE = DCF - DGE$ , the Trapezium  $ABGD = EGCF$ . Therefore, by *Ax.* 7,  $ABGD + BGC = EGCF + BGC$ , i. e. the Parallelogram  $AD = EBCF$ .

*Corol.* 1. Parallelograms BCDA and BCFE, upon equal Bases BC and GH; within the same Parallels AF and BC, are equal. For the Lines BE and CH being drawn, by this *Theorem* 7, the Parallelogram  $BCDA = BCFE = GHFE$ . *Fig. 67.*

*Corol.* 2. Triangles BCA and BCD, upon the same Basis BC, and within the Parallels BC and EF, are equal, the Line BE being drawn parallel to BA, and CF to BD, it follows by this *Theorem*, that the Parallelogram  $BCAE = BCFD$ . Wherefore, by *Axiom* 12,  $\frac{1}{2}BCAE = \frac{1}{2}BCFD$ . *Fig. 68.*

$\triangle BCAE = \triangle BCFD$ , *i. e.* by Cor. 5, Theorem 5, the Triangle  $BCA = BCD$ .

Fig. 69.

Corol. 3. Triangles  $BCA$  and  $EFD$  on equal Bases  $BC$  and  $EF$ , and within the same Parallels  $GH$  and  $BF$ , are equal. For,  $BC$  being drawn Parallel to  $EF$  and  $EH$  to  $FD$ , it follows, by Corol. 2, of this Theorem, that the Parallelogram  $BCAG = EFHD$ . Wherefore, by Axiom 12,  $\triangle BCA = \triangle EFD$ . *i. e.* by Corol. 1, Prop. 5, the Triangle  $BCA = EFD$ .

Fig. 70.

Corol. 4. A Parallelogram  $ABCD$  is the Double of a Triangle  $BCE$ , upon the same Basis  $BC$ , and within the same Parallels  $AE$  and  $BC$ . For,  $AC$  being drawn, the Triangle  $BCA = BCE$ , by Corol. 2, of this Theorem. Therefore,  $\triangle BCA = \triangle BCE$ . But by Corol. 1, Theorem 5, the Parallelogram  $ABCD = 2\triangle BCA$ ; therefore  $ABCD = 2\triangle BCE$ . *Axiom 3.*

Fig. 71.

Corol. 5. Parallelograms  $ABCD$  and  $EDEF$ , if they have the same Altitude  $ED$ , are one to another as their Bases  $BC$  and  $CF$ ; and, if they have the same Basis  $CD$ , then they are one to another as their Altitudes  $BC$  and  $CF$ . For, in both Cases; (*viz.* whether  $BC$  and  $CF$  be taken for the two several Bases, and  $ED$  for the common Altitude; or whether  $CD$  be taken for the common Basis, and  $BC$  and  $CF$  for the two different Altitudes)

Parallelograms ABCD and CDEF be Rectangles, are by *Defin. 41*, made respectively of  $BC \times CD$ , and  $CF \times CD$ ; consequently; being the Products of Quantities BC and CF equally multiplied, namely, by one and the same it follows by *Axiom 11*, that  $CD : CDEF :: BC : CF$ . And it be thus demonstrated, that rectangular Parallelograms are one to another as Bases, if their Altitudes be the same; or, as their Altitudes, if their Bases be the same, it follows, that all rectangular Parallelograms will be so farasmuch as they are equal (by Theorem the 7th) to Rectangles upon the same Basis, and within the same Altitudes. Wherefore, Parallelograms in which the Bases are equal, and the Altitudes being thus proportional, if they have the same Altitude or Basis; it follows from *Corol. 1*, of this Theorem, they must likewise be proportional, and have equal Altitudes or Bases.

*Prop. 6.* Triangles BCD and CDE, if Fig. 72 have the same Altitude CD, are one to another as their Bases BC and CE; if they have the same Basis CD, they are one to another as their Altitudes BD and DE. For, by *Corol. 4*, of this Theorem, the Triangle BCD, is equal to the Parallelogram ABCD; and the Triangle CDE, equal to Half the Parallelogram

lelogram CDEF. But, by *Corol.*  
 $ABCD : CDEF :: BC : CF$ . There  
 by *Axiom* 13,  $\frac{1}{2}ABCD : \frac{1}{2}CDEF ::$   
 $CF$ , i. e.  $BCD : CDF :: BC : CF$ .  
 hence it follows from *Corol.* 3, of  
 Theorem, that Triangles, which  
 equal Altitudes, will be to one ano  
 as their Bases; and Triangles, wh  
 have equal Bases, will be one to ano  
 as their Altitudes.

## U S E.

I.  
 The Foun-  
 dation of  
 Surveying  
 without  
 Protracti-  
 on, demon-  
 strated.

Besides other Uses of this 7th Theorem and its Corollaries, it is of great Use in Surveying or Measuring Land the best Way of doing which, (viz. taking the Perpendicular in the Field and so quite omitting Protraction,) in a manner wholly depend on this Theorem and its Corollaries.

2.  
 The measu-  
 ring of Ob-  
 liquangu-  
 lar Paral-  
 lelograms,  
 deduced  
 from that  
 of Rectan-  
 gles.

For the Area or Content of any Rectangle being found (as has been above serv'd in *Defin.* 41.) by multiplying of its contiguous Sides together, it follows from this Theorem it self, and *Corol.* 1, that the Content of any obliquangular Parallelogram may be found by making a Rectangle upon the length of an equal Basis, and within the same Parallels. For by the said Theorem its *Corol.* 1, it is shewn, that the Co



The obliquangular Parallelogram, will be the same with the Content of a Rectangle so made.

And in like manner from *Corol. 4.* of <sup>3.</sup> Theorem, is deduced the Method of finding the Content of any Triangle, viz. <sup>The Measuring of Triangles, deduced from that of Parallelograms.</sup> Compleating the Parallelogram, of which the Triangle given is an Half; if the said Parallelogram be obliquangular, by making a Rectangle upon the same or an equal Basis, and within the same Parallels, as is the obliquangular Parallelogram. For, by *Corol. 4.* it is shewn, that the Content of a Triangle, is equal to Half the Content of a Parallelogram upon the same Basis, and within the same Parallels.

The Method of Measuring (or finding the Content of) any Triangle being thus shewn, thence is deduced the Method of Measuring any Trapezium. For a Trapezium may be resolv'd into two Triangles by a Diagonal; the Contents of which two Triangles being found, the Content of the whole Trapezium is found, being the Sum of the Contents of the two Triangles. Thus, *Fig. 73.* the Trapezium ABCD, is by the Diagonal resolv'd into two Triangles ABC and CDA. And the Content of the Triangle ABC, is equal to Half the obliquangular Parallelogram ABHC, (being both

Q

<sup>4.</sup> The Measuring of a Trapezium, deduced from the Measuring of Triangles.

*Fig. 73.*

both on the same Basis AC, and w  
the same Parallels AC and BH,) consequently is equal to Half the Re  
gle KBHG (=Parallelogram ABHC  
having the same Basis BH, and B  
within the same Parallels AC or AG  
BH). So the other Triangle AC  
 $\frac{1}{2}$  Parallelogram ACDE =  $\frac{1}{2}$  Parallelo  
DEFL. Wherefore, the Content of  
whole Trapezium ABCD = ABC +  
=  $\frac{1}{2}$  ABHC +  $\frac{1}{2}$  ACDE =  $\frac{1}{2}$  KBHG +  $\frac{1}{2}$  D

5.  
The short  
Method  
used in  
Practice to  
do this.

But now the Content of a Rect  
KBHG, (or DEFL) being found by  
tiplying KB into BH, (or LD into L  
and the Triangle ABC, being  
 $\frac{1}{2}$  KBHG, (or the Triangle ACD,  
only  $\frac{1}{2}$  DEFL.) it follows, that the  
tent of the Triangle ABC, must be  
to  $KB \times \frac{1}{2} BH$ , or  $\frac{1}{2} KB \times BH$ , (and lik  
ACD =  $LD \times \frac{1}{2} DE$ , or to  $\frac{1}{2} LD \times DE$ ).  
the Line KB being a Perpendicular  
fall from B, the Angle (of the Tri  
ABC) opposite to the Basis AC, an  
said Basis AC, being equal to BH  
other contiguous Side of the Rect  
KBHG, hence it comes to pass, th  
is not necessary in Practice to draw  
ther the whole obliquangular Par  
gram ABHC, or the whole Rect  
KBHG, which are the Doubles of  
Triangle ABC, but) in Practice it  
sufficient to know the Length of the

pend

pendicular KB, in order to know the Content of the Triangle ABC: Forasmuch as  $KB \times \frac{1}{2} AC (= KC \times \frac{1}{2} BH)$  is the Content of the said Triangle. And in the same manner in the Triangle ACD, being given the Length of the Basis AC, it is sufficient to know the Length of the perpendicular DL, in order to know the Content of the Triangle ACD. Forasmuch as  $LD \times \frac{1}{2} AC (= LD \times \frac{1}{2} DE)$  is the Content of the said Triangle, as has been already shown.

Since more, forasmuch as the Diagonal AC of the Trapezium ABCD is the common Basis, both of the Triangle ABC, and also of the Triangle ACD, it is formed this universal Rule for finding the Content of a Trapezium, viz. the Diagonal AC of the Trapezium ABCD, being multiplied into the Sum of the two Perpendiculars KB and DL, in the two Triangles ABC and ACD, (or, which comes to the same, Half the Diagonal AC being multiplied into the whole Sum of the two Perpendiculars KB and DL,) will give the Content of the Trapezium ABCD.

Now there being no considerable Piece wanting, but what may be reduced to one of the fore-mention'd Figures, viz. a Triangle, Parallelogram, or Trapezium; it is evident from what has

Q 2

been

6.

*The Rules  
for finding  
the Area  
of a Trapezium.*

been said, how much the Art of Surveying depends on this 7th Theorem, and its four first Corollaries, in finding the Measure or superficial Content of given Ground or Piece of Land.

7.  
Of laying  
out Land.

And it is obvious, that on the Principles is founded in great Measure that other Part of Surveying, *viz.* laying out any Quantity of Land desir'd. To this the two last Corollaries of Theorem are particularly subservient. For thereby is demonstrated, how a Piece of Land (supposing it reducible to one or more Parallelograms or Triangles) being given, there may be laid out another Piece of Land, twice or thrice (or as big) namely, by doubling or tripling, (or as big) either the Basis, or the Altitude of the Parallelogram or Triangles, into which the Piece of Land given is reducible.

### THEOREM VIII.

Fig. 74.

A Line DE, drawn parallel to any Side BC of a Triangle ABC, will cut the other two Sides of the Triangle proportionally, *viz.*  $AD : DB :: AE : EC$

*Demon.* Draw the Lines CD and BD. Now the Triangle DEB = DEC, by Theorem 7. Wherefore, by Axiom 2,  $AE : DBE :: ADE : DEC$ . But  $DBE = DEC$ .



6, Theorem 7,  $ADE : DBE ::$   
 $DB$ , and by the same,  $ADE : DEC$   
 $(DBE) :: AE : EC$ . Therefore, by  
 9,  $AD : DB :: AE : EC$ .

U S E.

Among several other Uses of this  
 prem, it will be sufficient here to  
 serve, that on it is founded the Me-  
 of dividing a Line into any Number  
 parts requir'd, as also of finding a  
 Line proportional to three given,  
 a third proportional to two given,

THEOREM IX.

Angular Triangles  $ABC$  and  $DEF$ , *Fig. 75,*  
 the Sides about the equal Angles *& 76.*  
 portional, viz.  $AB : BC :: DE : EF$ ,  
 $BC : CA :: EF : FD$ , and  $CA : AB ::$   
 $DE$ .

*Dem.* The Angle  $E$  being given e-  
 to the Angle  $B$ , upon  $BC$  set off  
 $EF$ , and upon  $BA$  set off  $Bd = ED$ ,  
 draw the Line  $df$ , which will be e-  
 to  $DF$ , by *Corol. 1, Theorem 5.*  
 by Theorem the 8th,  $Bd : dA ::$   
 $BC$ . Wherefore, inversely  $Ad : dB ::$   
 $fB$ . Wherefore, by Composition  
 $+dB, i. e.) AB : dB :: (Cf + fB, i. e.)$

Q 3

CB

$GB : fB$ . Wherefore, alternately  
 $BC :: dB : fB$ , and (because by C  
 struction  $aB = DE$ , and  $Bf = EF$ .) th  
 fore, by Ax. 6,  $AB : BC :: DE$   
 Which was the first Particular to be  
 monstrated. And the rest may be  
 ved exactly after the same manner  
 making at the other two Angles A  
 of the Triangle ABC, a Triangle  
 to DEF.

*Corol. 1.* By Parity of Reason it  
 lows, that similar  
 or like Sectors  
 $ACB$  and  $aCb$ ,  
 have likewise the  
 Sides and Arches A  
 about the equal B  
 Angles proporti-  
 onal; viz.  $AC : BC :: aC : bC$ .

$AC : AB :: aC : ab$ , &c. For the  
 AB, and the right Line AB, are  
 the Measure of the Angle ACB,  
 likewise the Arch  $ab$ , and the right  
 $ab$ , are each the Measure of the  
 $aCb (=ACB)$ ; and therefore, the  
 Proportion as have the right Line  
 and  $ab$ , must the Arches AB and  $ab$   
 to one another, or any Thing else.

*Corol. 2.* Because  $AC : BC :: aC$   
 therefore, alternately  $AC : aC :: BC$



USE.

This Theorem the 9th is of very great  
in Practice. For on it is founded  
Practice of taking Heights and Di-  
ces, as also of Surveying, by Protra-  
For Protraction is nothing else in  
upshot, than drawing on Paper Tri-  
es, which, being equiangular to much  
er Triangles, have their Sides in the  
Proportion one to another, as are  
Sides of the larger Triangles, which  
represent. For Instance: It has  
observ'd, in Reference to *Fig. 45*,  
CA the Height of the Object, and  
the horizontal Distance from the Foot  
of the Object, to B the Eye or Place  
the Observer, and BC the Ray of  
on from C the Top of the Object, to  
the Eye of the Observer, make a Tri-  
e. Now the Height of any Object  
g always measur'd by CA a Perpen-  
ar, let down from C the Top of  
Object to A its Foot; hence it fol-  
that the Angle BAC is a right An-  
It has also been shewn in the Use  
Theorem 2, and 3, that the Angle  
is truly taken by the Quadrant,  
or the Length of BA, or the Distance  
the Observer from the Foot of the  
Object, it may (supposing the Foot of

I.  
As to  
taking  
Heights.

the Object to be accessible) be measured by Feet or Yards, &c. Now the Number of real Feet or Yards between Observer and the Object being known, they may be represented on Paper by Line BA, drawn of such a Length, as contain such a Number of Parts of Scale whence it is taken, as equal Number of real Feet or Yards as mention'd. Upon A, one End of Line BA is to be made a right Angle representing the right Angle made by perpendicular Height CA of the Object and the horizontal Line BA. And at the other End B of the Line BA, is to be made another Angle, representing and so equal to the Angle made by the horizontal Distance between the Observer and the Object, and CB the Ray of Vision. This being done, and Lines BC and AC being extended, they meet in C, there will be formed Triangle CAB on the Paper, which shall be equiangular to the Triangle made by the real Distance between the Observer and Object, the real Height of the Object, and the real Ray of Vision. Consequently, although the Sides of the Triangle on the Paper, be nothing more than so long as are the real Sides of that angle it represents, yet, by this Representation, the Sides of the Triangle on



er, will be *proportional* to the corresponding Sides of the real Triangle. and therefore, as BA is to CA measur'd the Scale, by which BA was drawn, will the real Distance between the Server and the Object, be to the real Height of the Object, measur'd by the same Measure (whether Feet or Yards, &c.) as the said Distance was measur'd. Namely, supposing the real Distance, represented by BA, to be forty Yards; and consequently, the Line BA the Paper to contain just forty Parts the Scale, by which it was drawn; if the Line CA on the Paper, being apply'd the same Scale, do's contain just thirty Parts thereof; thereby is shewn, that in the same manner, the real Height or Perpendicular of the Object, represented by CA, is 30 Yards.

And as the Use of this Theorem has been illustrated in Reference to taking Heights by Protraction; so it may be easily apprehended (by what has been said) in Reference to taking Distances. In Surveying by Protraction, these be done after the same manner as the former; namely, by drawing on Paper Angles, equiangular to the real Triangles, arising in taking Distances and in Surveying; the Sides of which Triangles on Paper, will by this Theorem have the same

2.

As to  
taking  
Distances.

same Proportion one to another, as the correspondent Sides of the real angles.

Lastly, The Corollaries annex to Theorem are of great Use in Mechan

## T H E O R E M X.

Fig. 80.  
—85.

Similar Planes are one to the other, the Squares of their homologous Sides

*Demon.* A Rectangle being made (according to *Defn.* 41.) by multiplying together two Lines, one of which denotes its Length, the other its Breadth; hence it comes to pass, that to know the Ratio or Proportion, which one Rectangle

Fig. 78,  
\* 79.

ABCD, has to the other EFGH, there must be known, not only the Proportion of AB the Length of one Rectangle to EH the Length of the other Rectangle, but also the Proportion of AD the Breadth of one, to EF the Breadth of the other. For as each Rectangle is made up of its own Length multiplied together, so the Ratio or Proportion of the said Rectangles one to the other, is compounded or made up of the Proportion of their Lengths and of the Proportion of their Breadths multiplied together. Thus, if  $AB = 3EF$ , and  $AD = 2EH$ ; then  $AB \times AD = 3EF \times 2EH$ , and  $ABCD = 6EFGH$ . In Words thus

Rectangle ABCD be thrice as long, twice as broad as the Rectangle GH; then the former will be (twice the Times, or thrice two Times, *i. e.*) Times as big as the latter. Which is presently represented to the very Eye, by 78, and 79, compar'd together. And like holding Good between all other Rectangles for the same Reason, hence Rectangles are said in general to be one another in a *compound* Ratio or Proportion of their contiguous Sides, *i. e.* in a Proportion as arises from the Proportion of their Lengths, and the Proportion of their Breadths multiplied together.

And, forasmuch as any similar Rectangles KLMN and OPQR, have (by *Defin.*) their contiguous Sides proportional,  $KL : LM :: OP : PQ$ ; and so alternely  $KL : OP :: LM : PQ$ , hence it follows, that if  $KL = 2OP$ , then likewise  $LM = 2PQ$ . And consequently to say, similar Rectangles are in a *compound* Proportion of their contiguous Sides, is other than to say, that they are in a *duplicate* Proportion, (*i. e.* as the Squares) of their homologous Sides. Namely, because  $KL = 2OP$ , and  $LM = 2PQ$ , therefore  $KL \times LM = 2OP \times 2PQ$ , *i. e.*  $KLMN = 4OPQR$ . And this is clearly represented to the very Eye, by Fig. 80, and 81, compar'd

Fig. 80,  
& 81.

compar'd together. Where, suppose KL to be four Foot long, and OP to be two Foot long, and LM to be six Foot long, and PQ to be three; it will follow, that the Area or superficial Content of KLMN, will be  $6 \times 4 = 24$  Feet Square, and the Area of OPQR, will be  $3 \times 2 = 6$  Feet square. And consequently KLMN will be four Times as big as OPQR, which is the Proportion of the Squares of their homologous Sides KL and LM and PQ. For  $KL^2 = 4^2 = 16$  Foot, and  $OP^2 = 2^2 = 4$ . In like manner  $LM^2 = 6^2 = 36$  Foot, and  $PQ^2 = 3^2 = 9$ . But  $24 : 6 :: 16 : 4 :: 36 : 9$ . i. e.  $KLMN : OPQR :: KL^2 : OP^2 :: LM^2 : PQ^2$ .

Case 1. And hence it follows, that similar Parallelograms are one to another as the Squares of their homologous Sides. Forasmuch as all obliquangular Parallelograms are (by Theorem 7, and its Corollary 1.) equal to Rectangles on the same equal Bases, and within the same parallels.

Fig. 82,  
★ 83.

Case 2. And hence it follows, that similar Triangles ABC and abc, are one to the other as the Squares of their homologous Sides BC and bc: Forasmuch as (by Corol. 4, Theorem 7,) all Triangles, th



are the Halves of Parallelograms upon the same Basis, and within the same Parallels. This is illustrated, *Fig. 82, 83*, compar'd with *Fig. 80*, and *81*. *Prop. 3*. Hence also it follows, that similar Polygons, as *Fig. 84*, and *85*, are likewise one to another, as the Squares *Fig. 84, & 85*. Their homologous Sides *AB* and *ab*, as *AB : ab*; Forasmuch as all such Polygons may be divided into an equal Number of similar Triangles, which shall be homologous to the whole Polygons. Lastly, Forasmuch as Circles may be divided on as similar Planes, and are divided into Semi-circles, which will therefore be likewise similar Planes, whose Diameters will be homologous Sides; the Circles (or Semi-circles) are also one to the other, as the Square of their Diameters, *Ex. gr.* a Circle, whose Diameter is four Inches, is four Times as big as a Circle, whose Diameter is two Inches. For  $4 \times 4 = 16$ , and  $2 \times 2 = 4$ ; and  $16 : 4 :: 4 : 2$ .

## U S E.

The great Use of this Theorem in Arithmetic, is abundantly evident from what has been already said in the Demonstration of it. For it thence appears, that hereon is founded the Method,

rhod, both of finding the Proportion of two (or more) similar Figures given, also of making two (or more) similar Figures in a Proportion assign'd.

## T H E O R E M XI.

**Fig. 86.** A Perpendicular AD being let fall from the right Angle BAC of a right-angled Triangle ABC, upon the Hypotenuse BC, the Triangles about the Perpendicular, *viz.* ADB and ADC, will be similar, both in Respect of the whole Triangle ABC, and also in Respect one of the other.

*Demon.* The Angle ADB is a right Angle by *Defin.* 20; and therefore, by *Axiom* 16, is equal to the given Angle BAC; and the Angle B is common both to the Triangle ABC, and the Triangle ADB; and consequently, by *Corol.* 3, Theorem 4, the Angle C is equal to the Angle DAB, in the Triangle ADB. Wherefore, the Triangles ABC and ADB are equiangular. In like manner it may be prov'd, that the Triangles ABC and ADC are equiangular; because the Angle C is common to both, and the Angle ADC and BAC are both Right Angles; therefore  $\angle DAC = \angle ABC$ . Now the Angles ADB and ADC, being both

ular to the Triangle ABC ; it thence  
ows, by *Axiom* 3, that they are equi-  
lar one to the other. And the seve-  
Triangles being prov'd to be mutual-  
equiangular ; it thence follows, by  
orem 9, that the Sides about the e-  
Angles are proportional. And con-  
sently, the said three Triangles are  
lar one to the other, by *Defin.* 39.

*Corol.* 1.  $BD : DA :: DA : DC$ .

*Corol.* 2.  $BD : BA :: BA : BC$  ; and  
 $CA :: CA : CB$ .

*Corol.* 3. Hence it follows, (by that  
ral Rule of Proportion, *viz.* the Pro-  
of the Extreams, is equal to the  
duct of the Means,) that  $BD \times BC =$   
; and  $CD \times CB = CA^2$ .

U S E.

In this Theorem, and its Corollaries,  
ounded the Method of finding a mean  
portional to two given Lines ; and  
one Method of finding a third Pro-  
portional to two given Lines. And on  
also depends the Demonstration of  
following noble, or most useful  
orem.

THEOREM.

## THEOREM XII.

Any Figure describ'd by the Hypotenuse, is equal to similar Figures describ'd by the other two Sides of the same angular Triangle.

Fig. 87.

*Case 1.* The (\*) Square BCEF of Hypotenuse BC, is equal to the Squares ABGH and ACKL of the other two Sides AB and AC, in the rectangular Triangle ABC. For, by *Ax. 2*,  $BC = BD + DC$ . Wherefore, by *Axiom 10*,  $(BC \times BC) = (BD + DC) \times (BD + DC) = BD \times BC + DC \times CB = BAq + CAq$ . *Corol. 3*, Theorem 11. Wherefore, by *Axiom 3*,  $BCq = BAq + CAq$ . Q. E. D.

Fig. 88.

*Case 2.* In like manner, the Rectangle BCEF describ'd by the Hypotenuse BC, is equal to the similar Rectangles ABGH and ACKL describ'd by the other two Sides AB and AC of the rectangular Triangle ABC. For, by Theorem 10, similar Figures are one to another, as the Squares of their homologous Sides, i. e.  $BCq : ABq :: ACKL : ABGH$ ; and likewise  $BCq : ACq :: ACKL : ACKL$ . Wherefore, by *Ax. 9*,  $BCEF : ABGH + ACKL :: BCq : ABq + ACq$ . But, (as is shewn, *Case 1*),

---

(\*) It was above noted in *Defin. 39*, that all Squares are similar Figures one to another.



$= AB^2 + AC^2$ ; therefore  $BC^2 =$   
 $GH^2 + ACKL$ . Q. E. D.

**U S E**

On this Theorem is founded the Me-  
 thod of Adding and Subtracting Squares,  
 and any other similar Figures. Likewise

on this Theorem is deduced the Me-  
 thod of finding the third Side of any  
 Angular Triangle, any two Sides being

known. For Instance; Let the Side AB *Fig. 89*

of the rectangular Triangle ABC be six  
 Foot long, and the Side AC be eight  
 Foot long. Hereby may be known the

Length of the other Side BC to be ten  
 Foot. For by this Theorem  $AC^2 + AB^2 =$

$BC^2$ , i.e. (in the Numbers of Feet)  $64 + 36 =$   
 $100$ . Wherefore  $BC = \sqrt{100} = 10$ . In

like manner, suppose the known Sides to  
 be AB six Foot, and BC ten Foot. Then

by this Theorem and *Ax. 8*, it follows,

$BC^2 - AB^2 = 100 - 36 = 64 = AC^2$ .

Therefore  $AC = \sqrt{64} = 8$ . Hence, sup-

posing AB to denote the Wideneſs of a

Wall, and AC the Height of a Wall,

it may be known how long a Lad-

d must be, to reach from the Out-

Side B of the Ditch, to the Top C

of the Wall. In ſhort, by Reason of its

Use, this Theorem is not only e-

ſteemed by Mathematicians one of the

**R**

moſt

most noble and useful Theorems in Geometry, but also it is said, that an Altar was offer'd to Apollo by the Discoverer of it, Pythagoras, (where is sometimes stiled the Pythagorick Theorem) in Gratitude for so useful a Discovery. With it I shall conclude this Chapter.

## CHAP. II.

Containing the more useful and Problems, which relate to Lines and Planes.

### PROBLEM I.

Fig. 90.

**T**O draw a right Line AB parallel to a given right Line BD, at a given Distance GE (or FH.)

Any two Points E and F of the Line CD being taken for Centers, and Compasses being opened to the Extent EG (or FH,) draw two Semicircles, draw the Line AB so, as just to touch (and not cross) both the said Semicircles, the said AB will be parallel to CD, the Lines EG and FH are by Construc-

the Rays of equal Circles, and therefore themselves equal by *Axiom* 17. And consequently, the Lines AB and CD, are distant one from the other at the points E, F, and G, H. And therefore, by both right Lines they are by *Defin.* equal distant all along, and therefore parallel by *Defin.* 34.  
 B. It is obvious, that in Practice it is not necessary to draw the Semicircles of equal Rays GE and HF, but it is sufficient to draw only the Arches (of the Semicircles) wherein the Line AB touches the Semicircles in the Points G and H, as *Fig. 91*; And the most ready *Fig. 91.* way to draw Parallels, is the mechanical way, by the Help of a *Parallel-rule*.

## PROBLEM II.

At the given Point B of a given Line *Fig. 92,* to make an Angle B equal to a given Angle *b*. Upon *b*, taken for a Center, draw the Arch *ac* taking off equal *ba* and *bc* of the Legs of the Angle *b*. With the same Extent of your Compass upon B describe the Arch of a Circle cutting DE at A. From A set off an Arch AC, equal to the Arch *ac*, and draw the Line BC. The Angle B, will be equal to the Angle *b*, by *Axiom* 18, *Defin.* 16; For both Angles are measured  
 R 2                      measur'd

measur'd by equal Arches of equal  
cles.

### PROBLEM III.

Fig. 94.

To bisect a given Angle BAC. **T**  
 $AD=AE$ ; and with the same Exten-  
your Compasses upon D and E, draw  
equal Circles crossing in F. Draw  
and EF, and AF. The Line AF will  
sect the Angle BAC. For AD and  
also DE and EF, are all (\*) by Con-  
struction Rays of equal Circles, and there-  
fore equal by *Axiom* 17, and AF is com-  
mon: Wherefore, by Theorem 5,  
Triangle ADF = AEF, and so the Angle  
 $DAF=EAF$ . And consequently,  
Angle BAC, made up of the Angles  
and EAF, is equally divided or bisected.

N. B. It is obvious, that in Practice  
it is not necessary to draw the Lines  
and EF; but they may be omitted  
being sufficient to draw the Line  
whereby the Angle BAC is bisected.

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(II) For to take  $AD=AE$ , is the same, as from  
Center) to draw Rays AD and AE of the same  
and DF and DE are Rays of equal Circles with  
AE.



# PROBLEM IV.

To make any Triangle requir'd, whe-  
Equilateral, Iſoſceles, or Scalenum :  
thereby to make any other right-  
Figure whatever, which ſhall be  
to a right-lin'd Figure given.

*Uſe 1.* To make an *equilateral* Trian-  
ABC. Fix one Foot of your Com-  
es on A, and opening the other to  
Extent of AB, draw a Circle ACBD.  
n keeping your Compaſſes at the  
e Extent AB, and fixing one Foot of  
n on B, draw the Circle BACE. From  
e Point, where the two Circles croſs  
another, draw the right Lines CA  
CB. Then, by *Deſin. 9*,  $AC=AB$   
C. Therefore, by *Axiom 3*,  $AC=$   
And conſequently, the Triangle  
is equilateral by *Deſin. 23*.

Fig. 95.

*Uſe 2.* The Baſis BC being given, to  
an *Iſoſceles* Triangle ABC, whoſe  
AB and AC ſhall be each of a given  
th *ab*. Open your Compaſſes to the  
th of *ab*, and then fixing one Foot  
draw the Circle BDA ; and then  
wing your Compaſſes (kept at the  
Extent *ab*) to C, draw the Circle  
From A the Point, where the  
Circles croſs, draw AB and AC,  
thereby will be made the Iſoſceles

Fig. 96.

R 3

ABC.

ABC. For AB and BC being Radii of Circles equal by Construction, they be themselves equal by *Axiom* 17, consequently, the Triangle ABC will be an Iſoſceles by *Defn.* 24.

Fig. 97.

*Case 3.* To make a *Scalenum* whose three Sides AB, BC, and AC shall be reſpectively equal to three given Lines *ab*, *bc*, and *ca*; any two of which given Lines being together longer than the third. (For otherwiſe they will not meet together to make a Triangle.) Draw a Line DE of a ſufficient Length, and thereon ſet off  $AB = ab$ , and  $BC = bc$ . Then upon A at the Extent of AF, draw a (Circle or) Semicircle AFC; and likewiſe upon B at the Extent of BG, draw another Semicircle BGC. From the Point C, where the (Circles or) Semicircles croſs, draw Lines AC and CB, and the Scalenum is finiſhed. For AB was taken equal to *ab*, and by *Defn.* 9,  $AC = AF$ , taken equal to *ca*, and alſo  $BC = BG$ , taken equal to *bc*. Wherefore, by *Axiom* 3,  $AC = ca$ , and  $BC = bc$ .

Fig. 98,  
& 99.

*Case 4.* To make any other right-angled Figure whatever, which ſhall be equal to a right-lined Figure given of the ſame Sort. Divide the Figure, ſuppoſed to be a Trapezium ABCD, *Fig.* 95, into two Triangles ABC and BCD. And then the

fore-going Cases, make Triangles respectively equal to the said Triangles, *Fig. 96*,  $abc = ABC$ , and  $bcd = BCD$ , which shall so join together. And hereby will be made the Trapezium  $abcd$   $BOD$ . And so of any Polygon.

*Case 5.* To make upon a given Line  $ab$ , *Fig. 98*, a similar Figure  $abcd$ , to any given Figure  $ABCD$ . Resolve the given Figure into Triangles; and then make the Angles  $abc = ABC$ , and  $bac = BAC$ , and  $acd = ACD$ , and  $cdb = CBD$ . The Figure will be a similar Trapezium, to the Trapezium  $ABCD$ . For, by Construction, and *Corol. 3*, Theorem 4, the Angle  $abc$ , is equiangular to  $ABC$ , and  $bac$  to  $BAC$ ; and therefore, by Theorem 9, the Side  $ab : bc :: AB : BC$ , &c, therefore the Trapezium  $abcd$  and  $ABCD$ , are similar by *Defin. 39*. It being evident, that in all the afore-mention'd the Point, where the Circles cross, is that from which two of the Triangle requir'd are to be drawn, therefore in Practice it is sufficient to draw only so much of the Circles, as will give the Point of Intersection. *Fig. 101*, the Arches  $ab$  and  $cd$  are sufficient to give the Point  $C$ , whence may be drawn  $CA$  and  $CB$ , in order to make the equilateral Triangle  $ABC$ , the same is to be understood in like

Cases relating to any of the following Problems.

PROBLEM V.

To divide a Line given into two, three, or more equal Parts.

Fig. 102.

Case 1. To divide a given Line  $AB$  into two equal Parts. With your Compasses open'd to the Extent of  $AB$ , and the Centers  $A$  and  $B$ , draw (the Arcs of) two equal Circles, on each Side of the Line  $AB$ , crossing at the Points  $C$  and  $E$ . The Line  $CE$  will bisect the given Line  $AB$ . For (as has been afore prov'd in Problem 3,)  $CE$  will bisect the Angle  $ACB$ , into  $ACD = BCD$ . And, because  $CA = CB$ , by Axiom 17, and  $CD$  is common, therefore, by Theorem 5, Corollary, the Triangle  $CAD = CBD$ , and so  $AD = DB$ ; and consequently, the given Line  $AB$  is bisected in  $D$ .

Fig. 103.

Case 2. To divide a given Line  $AB$  into more than two (suppose three) equal Parts. Draw the Line  $AC$  as you please, and thereon set off three equal Parts  $AD$ ,  $DE$ , and  $EF$ , of what Length you please. Draw  $FB$ , and parallel thereto draw  $EH$  and  $DG$ , and the Work is done. For, by Theorem 8,  $FD : DA :: GA : AB$ . Wherefore, by Ax. 9,  $(FD + GA) : DA :: (BG + GA) : BA :: AB : GA$ .



DA by Construction, therefore AB

B. It is obvious, that as to *Case* 1, sufficient in Practice (having drawn Arches crossing in C and D) to draw Line CD, whereby AB is bisected in So that all the other Lines AC, AE, BE, and DE, may be omitted.

# PROBLEM VI.

to draw a Perpendicular from, or to Point given.

*Case* 1. From a given Point C, to let a Perpendicular CD, upon a given AB. Upon the Center C, describe a Circle cutting AB in E and F. Then (by Problem 5, *Case* 1,) bisect The Point of Bisection will be D. Line CD being drawn, will be the Perpendicular requir'd. For CE and CF being drawn, and Rays of the same Circle ED and FD being divided into ED=FD, Triangles CDE and CDF, will be equally equilateral. And consequent- ly Theorem 5, the Angle CDE=CDF, and therefore, by Corol. 2, Theorem 5, both right Angles; and therefore CD is a Perpendicular to AB, by *Defin.*

Fig. 104.

*Case*

Fig. 105.

*Case 2.* Upon a given Point C, at a considerable Distance from each End A and B of a given Line AB, to erect a Perpendicular CF. Take  $CD = CE$ , upon DE make an equilateral Triangle DFE. The Line FC being drawn, shall be the Perpendicular sought. For Triangles DFC and EFC, are mutually equilateral by Construction, viz.  $DF = EF$ , and  $CD = CE$ , and FC common. Wherefore, by Theorem 5, the Angles DCF = ECF, and therefore, by Corollary Theorem 1, both right Angles, therefore FC is a Perpendicular to DE by *Defin. 29.*

Fig. 106.

*Case 3.* To erect a Perpendicular upon a given Point B, at (or very near) the End B of a given Line BE. Set one Foot of your Compasses upon B, open the other to what Extent BD you please, on that Side of BE as you wish, have the Perpendicular erected. Then upon D as a Center, describe an Arc or a Circle, which will touch the Line BE at B, and likewise touch or cross it (extended, if need be) again at C. Draw the Diameter CF, and then through B draw the Line AB, which will be the Perpendicular requir'd. For the Angle ABC being in a Semi-circle, is, by Corollary Theorem 6, a right Angle. And

ly AB is perpendicular to BE, by

20.

B. In Practice the Lines CE and (Fig. 101,) and FD and FE, (Fig. ) need not actually be drawn; not the Line BD in Fig. 103. And the ready Way to draw a Perpendicular Mechanically, or by the Help of a *re-rule*.

## PROBLEM VII.

to find a mean or third Proportional to two given Lines; or to find a fourth proportional to three given Lines.

*Case 1.* Two Lines AE and EB being *Fig. 107.*

to find a mean Proportional EF. Take the whole AB for a Diameter; and being bisected, the Point C of Division will be (by *Defn. 9.*) the Center of the Circle, whereof AB is a Diameter. Upon C, therefore describe the Circle AFB; and upon E erect a Perpendicular EF, touching the Circumference in F, which will be the mean Proportional sought. For the Lines AF and FB being drawn, will together with AE make a Triangle ABE, whose Angle at E will be a right Angle, by *Corol. 2,* *Prop. 6.* Wherefore, by *Corol. 1,* *Prop. 11,*  $AE : EF :: EF : EB.$

*Case*

## The Young Gentleman's

**Fig. 108.**

*Case 2.* Two Lines AB and AD given, to find a third Proportional Join BD, and from AB extended,  $BC=AD$ ; then draw CE parallel to BD and extend AD to E. The Line AE will be the Proportional sought. by Theorem 8,  $AB : BC :: AD : DE$ . But, by Construction  $BC=AD$ . Therefore, by *Ax. 6*,  $AB : AD :: AD : DE$ .

**Fig. 109.**

*Case 3.* Three Lines DE and EF, DG being given, to find a fourth proportional GH. Join EG, and draw FH parallel to EG, and extend DG to H. It follows by Theorem 8, that  $DE : DG :: EF : GH$ .

## P R O B L E M VIII.

To find the Center of a Circle whole Circle being given, or an Arc of it, or only three Points in it.

**Fig. 110.**

*Case 1.* To find the Center C of a Circle CABDE, the whole Circle being given. Draw in the Circle a right Line AD as you please, which bisect. bisecting Line BE bisect again. Point of its Bisection C, will be the Center sought. If it be denied, let the Center be F, lying out of BE, (for it can't lie in it, since BE is every where equally divided but in C,) and draw FA and FD. If F be the Center, the



9,  $FA=FD$ , and  $AD$  was bisected into  $AG=GD$ , and  $FG$  is common. Therefore, by Theorem 5, the Angle  $FAG=FDG$ , and therefore, both right Angles, by Corol. 2, Theorem 1; and therefore, by Axiom 16,  $FGD=CGD$ . being a right Angle, as made by bisecting Line  $BE$ , which, as is said, Problem 5, Case 1, is always perpendicular to the bisected Line  $AD$ . This is contrary to Axiom 1,  $CGD$  only a Part of the Angle  $FGD$ .

2. To find the Center  $C$  of a Circle  $ABD$ , only an Arch  $ABD$  of it being given. Draw within the Arch two Lines  $AB$  and  $BD$  as you please, and bisect. The two bisecting Lines  $EC$  and  $FC$  being extended within the Circle will meet in  $C$  the Center sought. It follows from Case 1, that the Center must be both in  $EC$  and in  $FC$ ; and therefore it can be only in  $C$  the Point of their Intersection or Meeting. The Center  $C$  being found, it is obvious, that the whole Circle  $CABDG$  may be completed, by fixing one Foot of your Compass on  $C$ , and opening the other to any Point of the Arch given  $ABD$ , and describing the other Arch  $DGA$ .

Fig. 111.

3. To find  $C$  the Center of a Circle, only three Points  $A$  and  $B$ , and  $D$  being given. It is evident, that  $A$  and  $B$  being joined, and  $C$  the Center of the Circle  $ABD$  found, the same Center  $C$  will be found by joining  $B$  and  $D$ , and finding the Center of the Circle  $BCD$ .

Fig. 112.

and B, and D, are in effect no other the extream Points of the Lines AB BD in *Case 2*. Wherefore, the said P being join'd by drawing AB and the Work in this *Case* is exactly the same as in *Case 2*. Hence also (by the Work) is obvious, that any three Points given, not lying in a straight Line, a Circle may be drawn through them, the Center of which Circle is to be found as is here directed.

### PROBLEM IX.

To make a Square, or any other of Parallelogram desir'd.

*Fig. 113. Case 1.* To make a Square ABCD Side AD being given. Raise two Perpendiculars AB and DC, each equal to AD, and draw BC; and the Square ABCD will be made. For AB and DC being Perpendiculars by Construction Angles A + D will be equal to two Right Angles by *Defin. 20*; and therefore AB and DC will be parallel by *Corol. 3*. Theorem 3. And because AB and DC were each equal to AD, they are therefore by *Defin. 3*, equal one to the other; and consequently BC is equi-distant to AD, so parallel thereto by *Defin. 34*. Therefore, by *Defin. 35*, ABCD is a Parallelogram, and also  $BC = AD$ .

the Angles  $A$  and  $D$  are right  
by Construction, therefore the  
Angles  $B$  and  $C$  are also right Angles by  
1, Theorem 3. And all the four  
Angles being thus Right, the Parallelo-  
gram  $ABCD$  is a Square by *Defin.* 29.

2. To make an oblong Parallelogram *Fig. 114:*  
desir'd  $ABCD$ , two contiguous  
Lines  $AB$  and  $AD$  being given. Raise  
Perpendiculars  $AB$  and  $DC$ , each of  
the same Length with the given Lines,  
draw  $BC$ , and it is done. For (as  
in a Square) the Angles  $A + D = 2$   
by Construction, and therefore  
the Lines  $AB$  and  $DC$  are parallel by *Corol.* 9,  
Theorem 3. Likewise  $AB = DC$  by Con-  
struction, and therefore  $AD$  and  $BC$  pa-  
rallel by *Defin.* 34, and so by *Defin.* 35,  
the Figure  $ABCD$  is a Parallelogram,  
and  $AB = DC$ . And the Angles  $A$  and  
 $D$  being each Right, their intern and op-  
posite Angles  $B$  and  $C$  will also be each  
Right, and consequently, the Parallelo-  
gram  $ABCD$  will be an Oblong by *Defin.*

3. To make a Rhombus desir'd *Fig. 115:*  
one Side  $AD$ , and one oblique  
Line  $E$  being given. Upon  $A$  make an  
Angle  $AD = E$ . Then from  $A$ , set off  
the Length of  $AD$ . Then, by Problem 1, draw,  
at Distance of  $AB$ , the Line  $BG$  pa-  
rallel to  $AD$ , on which set off  $BC = AD$ ,  
and

and draw DC. The Rhombus ABCD is made. For by Construction  $AD = BC$ , and AD parallel to BC. Also because  $AD = BC$ , therefore AB is parallel to DC. And consequently, by Definition, ABCD is a Parallelogram, and AB = DC. Wherefore, the Figure ABCD having four Sides equal, will be by Definition a Rhombus, and forasmuch as one of its Sides, and it has also the Angle  $A = E$  by Construction, it is the Rhombus as was requir'd.

Fig. 116. Case 4. To make a Rhomboid ABCD, two contiguous Sides AB, AD being given, and also one of the Angles E. Upon A make an Angle  $F = E$ . From AF, set off AB. To the Distance of AB, draw BG parallel to AD; on which set off  $BC = AD$ , draw CD. The Rhomboid is made by Construction, BC is parallel to AD; and consequently, by Definition, AB is parallel to CD. And so by Proposition 35, ABCD is a Parallelogram, and  $AC = CD$ . Wherefore, AC being drawn will follow, by Cor. 2, Theorem 1, the Angle  $BCA = CAD$ , and  $CAB$ . Wherefore, by Axiom 3,  $ACD = CAD + CAB$ , i. e.  $BCD = E$ . And after the same manner it is prov'd, that the Angle ABC = E. Wherefore, the Parallelogram ABCD is a Rhomboid.



four Sides, and four Angles; where  
only the two opposite Sides and An-  
gles are equal, (viz.  $AB=CD$ , and  $BC$   
 $AD$ , and the Angle  $A=C$ , and  $B=D$ .)  
It is a Rhomboid by *Defn.* 32. For  
N. B. It is obvious, that the Reason,  
in making a desir'd Square or Ob-  
long, no Angle is requir'd to be given,  
because these two Sorts of Parallelo-  
grams are always rectangular. Likewise  
Reason, why in making a desir'd  
Square or Rhombus, there needs be giv'n  
only one Side, is, because these Sorts  
of Parallelograms are equilateral. It is  
observable, that in Practice it is suf-  
ficient, two contiguous Sides of the Par-  
allelogram being drawn, viz.  $AB$  and  
 $AD$ , to set one Foot of your Compasses  
open'd to the Length of  $AD$  upon  $B$ ,  
to make an Arch at  $C$ ; and then set  
one Foot of your Compasses (open'd  
to the Extent of  $AB$ ) on  $D$ , to make ano-  
ther Arch at  $C$ ; and from  $C$  to draw the  
Lines  $CB$  and  $CD$ .

## PROBLEM X.

To change Figures of one Sort, into  
Figures of another Sort; or, to  
make Figures equal to Figures of another

**Fig. 117.** *Case 1.* To make a rectangular Parallelogram BCDA, equal to an oblique Parallelogram BCEF. Upon Side BC of the Parallelogram BCEF rect. the Perpendiculars BA and CD, produce EF till it meets BA. The figure BCAD will be a Rectangle by same Proof as is us'd, *Case 1*, and Problem 9. And it will also be equal BCEF, by Theorem 7.

**Fig. 118.** *Case 2.* To make a Parallelogram ECGF, which shall be equal to a Triangle ABC, and shall have an Angle ECG, equal to a given Angle D. Draw AG parallel to BC, and make the Angle BCG = D. Then bisect BC, and from the Point of Bisection, draw EF parallel to CG. Thereby will be made the Parallelogram ECGF = ABC, with an Angle ECG = D. For, there will be constructed the Angle ECG (BCG) and (AE being drawn) the Triangle ECG = BAE + EAC = (by Corol. 2, Theorem 7, and Ax. 7.) 2EAC = Parallelogram ECGF, by Corol 4, Theorem 7.

**Fig. 119, 120, & 121.** *Case 3.* To make a Parallelogram FHLM, (Fig. 121,) which shall be equal to a given Triangle B, (Fig. 119,) shall have an Angle MFH, equal to a given Angle C, and also a Side FM equal to a given Line A, (Fig. 120.) (by the preceding Case) the Parallelogram

in  $DEFG$  = to the Triangle B, and  
 ing the Angle  $GFE = C$ . To  $GF$   
 on in a right Line  $FA = A$ . Thro'  
 draw  $IL$  parallel to  $EF$ , and extend  
 to  $I$ . Draw  $IF$ , and extend it till it  
 meets in  $K$  with  $DG$  extended. Draw  
 parallel to  $GH$ , and extend  $EF$  to  $M$ .  
 Figure  $FHLM$  will be the Parallelo-  
 gram requir'd. For, by *Corol. 6*, Theor.  
 the Parallelogram  $FHLM = FGDE =$   
 angle B by Construction; and the  
 $MFH$  (by Theorem 2,)  $= GFE =$   
 Construction.

*Case 4.* To make a Parallelogram  $abcd$ , *Fig. 122;*  
 125,) which shall be equal to any 123,  
 a right-lin'd Figure, v. g. the Tra- 124, &  
 m  $ABCD$ , *Fig. 122*, and shall have 125.  
 angle  $bdc$ , equal to a given Angle  $E$ ,  
 Side  $bd$  (or  $ac$ ) equal to a given  
 $fg$ . Resolve the Trapezium  $ABCD$ ,  
 the Triangles  $ABD$  and  $CBD$ . And  
 by *Case 3*, make the Parallelogram  
 (Fig. 123,) equal to the Triangle  
 with the Angle  $GFI = E$ , and the  
 $GO = fg$ . In like manner make the  
 elogram  $FOLK$ , (Fig. 124,) equal  
 other Triangle  $CBD$ , with the  
 $GO = fg$ , and the Angle  $GFL = E$ .  
 by *Case 4*, Problem 2, make (Fig.  
 the Parallelogram  $acfg = FOIH$ ,  
 joining thereto  $fgbd = FOKL$ . The  
 parallelogram  $abcd$ , (Fig. 125,) will be  
 equal

equal to the given Trapezium ABC  
 Fig. 122. For  $ABCD = \text{Triangle } ABC$   
 $+ CBD = \text{Parallelograms } FGIH = FG$   
 $= acfg + fgbd$  by Construction. And  
 Angle  $fgc$  (or  $bdg$ )  $= OFI = E$  by C  
 struction.

Fig. 126,  
 127.

Case 5. To make a rectangular Tri  
 gle LAM, (Fig. 127,) equal to (any  
 ven Polygon describ'd about a Circle  
 for Instance,) the Pentagon, Fig. 126.  
 Draw a Line AM, (Fig. 127,) equal  
 all the five Sides of the Pentagon,  
 $ac + ce + eg + gi + ia$ . On one End  
 hereof erect a Perpendicular LA,  
 to the Ray  $lb$  of the Circle, Fig. 127.  
 Draw the Line LM. The rectang  
 Triangle LMA, is equal to the Pent  
 given. For, the Triangles ABL,  
 CLD, &c. are equal to the Tri  
 $abl, bld, cld, \&c.$  by Construction.  
 the Triangle  $ALM = ALB + BLC +$   
 $\&c.$  (by Corol. 2, and 3, Theore  
 $= alb + blc + cld, \&c. = \text{Pentagon}$   
 Axiom 2.

Fig. 128,  
 & 129.

Case 6. To make a rectangular  
 gle ABC, which shall be equal to  
 ven Circle CDEF. It is found, that  
 Circumference of a Circle is to the  
 meter, much as 22 to 7. When  
 draw (Fig. 128,) the Line BC  
 Times as long as the Diameter D  
 a 7th Part more. Upon B erect



pendicular BA, equal to the Ray CF.  
Line AC being drawn, the Triangle  
ABC will be equal to the given Circle.  
as it has been shewn in Case 5, that  
a Polygon is equal to a rectangular Tri-  
angle, one of whose Legs about the right  
Angle, is equal to the Perimeter of the  
Polygon, the other to the Ray of a Circle  
in which the Polygon is described; so  
by the same Reason it follows, that a Circle  
(which may be look'd on as a Poly-  
gon) is equal to a rectangular Triangle,  
of whose Sides about the right Angle,  
one is equal to its Circumference, the other  
to its Ray.

Case 7. It is obvious, that having made  
a Triangle equal to a Polygon or Circle;  
it may be made a Parallelogram (by  
Prop. 4, &c.) equal to the said Triangle,  
and so equal to the said Pentagon or  
Circle. Thus, (Fig. 129,) the Paralle-  
logram CDEF = Triangle ABC = Circle  
ABC, Fig. 128.

Case 8. To make a Square ABCD, equal to a given rectangular Parallelogram  
Extend any Side  $de$  of the given  
Parallelogram, and on it being extended  
take  $ce = cb$  the contiguous Side of the  
Parallelogram given. Bisect  $de$ , and on  
the Point of Bisection draw a Semi-  
circle. Upon  $e$  raise a Perpendicular  $cA$ ,  
touching the Semi-circle in A. This Per-  
pendicular  $cA$  (or  $CA$ ) will be a Side of

Fig. 130.

the Square requir'd. Wherefore, let  $CD$  (or  $cD$ )  $= CA$ ; and then with same Extent of the Compasses set upon  $C$  and  $D$ , make two Arches crossing in  $A$ . Draw the Lines  $BA$  and  $BD$ , and there will be made a Square  $ABCD =$  Parallelogram  $abcd$ . For, by *Corol. 1*, Th. 9,  $CA$  is a mean Proportional between  $Ce$  and  $Cd$ . Wherefore,  $Ce \cdot Cd = CA^2$ . And  $ABCD = CAq$ , by *Case 1*, Prop. 9.

*Case 9.* Hence it is obvious how to make a Square equal to any oblique Parallelogram given; namely, by making first a rectangular Parallelogram equal to the given Rhombus or Rhomboid; and then by making a Square equal to the said rectangular Parallelogram. Thus, *Fig. 128*, the Rhomboid  $ABCD$  being given, by *Case 1*, of Problem 10, the Rectangle  $ABEF$  is made equal thereto; and then, by *Case 1* of this same Problem, the Square  $BCGH$  is made equal to the Rectangle  $ABEF$ ; consequently to the Rhomboid  $ABCD$ .

*Fig. 132.* *Case 10.* In like manner it is obvious that a Square may be made equal to any Triangle given  $ABC$ , by making the Rectangle  $CDEF$  equal to the Triangle, according to *Case 2*, of this Problem. Then the Rectangle  $CDEF$  equal to the Triangle, and then (by *Case 7*), a Square  $CGHI$  equal to the said Rectangle.

Case 11. It is also obvious on the same  
 point, that a Square may be made e-  
 qual to any given Trapezium, Polygon,  
 Circle, viz. by making first (as has  
 already shewn) a Rectangle equal to  
 said Trapezium, Pentagon, or Circle;  
 then, by Case 7, of this Problem,  
 making a Square equal to the said Rectan-

Thus, Fig. 129, the Square FGH  
 rectangle DCFE = Triangle ABC =  
 rectangle CDFE, Fig. 128. And this is  
 the Squaring of the Circle; though  
 it must be observ'd, that there is not so  
 an Equality between such a Square  
 and its Circle, as between other Figures  
 and such their Squares. The Reason  
 thereof is, because the Proportion of  
 Circumference to the Ray, is not ex-  
 actly as 22 to 7. Nor is there yet any  
 geometrical Way discover'd, whereby a  
 Line may be found, which shall be  
 exactly equal to the Circumference of a  
 Circle.

Case 12. Lastly, To make an Oblong  
 (Fig. 133.) equal to a given Square  
 D. Upon AB lengthened, set off  
 2AB. Bisect BC, and draw EG par-  
 allel to BF, and FG parallel to BE.  
 $AB \cdot BC = 2AB \cdot \frac{1}{2}BC = BF \cdot BE.$

## PROBLEM XI.

To add or subtract Figures to, from one another.

Fig. 134,  
compar'd  
with Fig.  
87.

*Case 1.* To add two (or more) Squares ABGH and ACKL together; or (which comes to the same) to make a Square BCEF, which shall be equal to two of Squares ACKL and ABGH. Draw Lines AZ and AX perpendicular one to the other, and so making a right Angle ZAX. Upon AZ, set off AC the given Side of the Square ACKL, and likewise upon AX, set off AB the given Side of the other Square ABGH; and draw BC. By Theorem 12,  $BC^2 = AC^2 + AB^2$ , that is, the Square BCEF, (Fig. 87,) = ACKL + ABGH. If to these two Squares

Fig. 135.

would add also a third MNOP, (Fig. 135,) then upon AZ, set off AP = MN, and upon AX, set off AR = MN, draw PR. By Theorem 12,  $PR^2 = (BC^2) + AR^2$  (MN<sup>2</sup> or MNOP.)

Fig. 134,  
compar'd  
with Fig.  
88.

*Case 2.* To add two (or more) Figures, viz. Rectangles ACKL and ABGH together; or to make a Rectangle BCEF, which shall be equal to two similar Rectangles ABGH and ACKL. The construction is exactly the same as in *Case 1*, and is illustrated by the same Fig. 131, compar'd with Fig. 88.



*Case 3.* To subtract a Square  $ABGH$ , from a greater Square  $BCEF$ . Upon  $DA$ , off  $BD=BC$  the Side of  $BCEF$ , and the Side of  $ABGH$ . Then taking  $B$  the Center, describe a Semicircle; erect a Perpendicular  $AC$ , which meet the Semicircle in  $C$ . Then being drawn, there will be by Theor.  $BCq=ABq+ACq$ . Wherefore, by *Prop. 8*,  $BCq-ABq=ACq$ .

*Fig. 136, compar'd with Fig. 87.*

*Case 4.* To subtract a similar Figure, suppose a Rectangle  $ABGH$ , from a greater similar Rectangle  $BCEF$ . The Praxis is the same as in *Case 3*, and consequently is illustrated by the same *Fig. 133*, compar'd with *Fig. 88*.

*Fig. 136, compar'd with Fig. 88.*

*Case 5.* If the similar Figures to be added or subtracted be not Squares or other Parallelograms, but Triangles, Trapeziums, or Polygons; then they may be turn'd into Parallelograms by *Problem 10*, and after that added or subtracted, as been shewn.

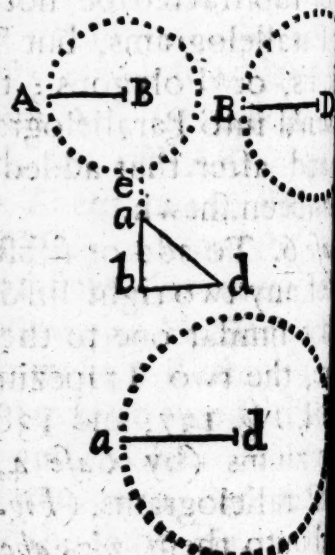
*Case 6.* To add or subtract the Quantity of any two right-lin'd Figures, which are not similar one to the other, for Instance, the two Trapeziums  $ABCD$  and  $EFGH$ , *Fig. 137*, and *138*. Turn the said Trapeziums (by *Case 4*, *Problem 10*) into Parallelograms, (*Fig. 139*, and *140*.) and add them, viz.  $abcd=ABCD$ , and  $efgh=EFGH$ . Then Parallelogram  $acgh$ , (*Fig.*

*Fig. 137, 138, 139, 140, & 141.*

(Fig. 141,) = Parallelograms  $abcd$  + Trapezium  $ABCD$  +  $EFGH$ . And parallelogram  $efbd$  = Parallelograms  $efgb$  = Trapezium  $ABCD$  -  $EFGH$ . And further, if you'd have the Parallelogram, which is equal to the two Trapeziums, to be a Square; having the Parallelogram  $acgb$  equal to them has been shewn; there may be made (Case 8, Problem 10,) a Square  $efgd$ , (Fig. 141,) = Parallelogram  $acgb$ . And in like manner may a Square be made equal to  $efbd$ , or Trapezium  $ABCD$  -  $EFGH$ .

Case 7. To add or subtract any or more Circles together. And first add two Circles together, or (which is the same) to make a Circle equal to two given Circles.

Draw the Line  $ab$ , equal to  $AB$  the Ray of one of the given Circles, and perpendicularly thereto draw the Line  $bd = BD$  the Ray of the other given Circle. Then draw  $da$ , which will be the Ray of



Circle equal to the two given Circles,  
by Theorem 12,  $adsqu. = bd\ squ. +$   
 $qu.$  And, by Theorem 10, *Case 4*,  
Circles are one to the other as the Squares  
of their Diameters; and consequently,  
the Squares of their Semi-diameters or  
Radii. Wherefore, the Circle  $ad =$  Cir-  
 $cle AB +$  Circle  $BD$ . *Secondly*, if you'd  
subtract the Circle  $BD$  from the Circle  $ad$ ,  
it would make a Circle  $AB$ , which  
would be so much less the Circle  $ad$ , as  
the Content of the Circle  $BD$ : Draw  
Line  $bd = BD$ , and upon one End  $b$   
draw a Perpendicular  $be$ . Set off the Ex-  
cess  $ad$  from  $d$ , to the Perpendicular  $be$ .  
The Line  $ab$  is the Ray of the  
Circle sought.

## PROBLEM XII.

To increase or diminish a given Figure  
in a given Proportion.

*Case 1.* To double (or triple, &c.) a Fig. 142.

Triangle  $abc$ ; or, to make a Tri-  
angle  $ABC$ , which shall be twice (or  
three, &c.) as big as a given Triangle  
 $abc$ . Double (or triple, &c.) any one  
Side of the Triangle  $abc$ ; or, which  
is the same, make  $BC = 2bc$ , and on  $BC$   
describe the Triangle  $ABC$ , within the same  
Angles  $Aa$  and  $bC$ , as the Triangle  $abc$   
within. The Triangle  $ABC = 2abc$ ,  
by

by *Corol. 6*, Theorem 7; It is obvious that if you'd have a Triangle thrice as big as  $abc$ , then you must take for Base a Line thrice as long as  $bc$ , and work as afore.

**Fig. 142.** *Case 2.* To make a Triangle  $abc$ , which shall be Half of a given Triangle  $ABC$ . Draw  $bc = \frac{1}{2}BC$ , and thereon make Triangle  $abc$ , within the same Parallels  $Aa$  and  $bC$ , and  $ABC$  is contain'd within. The Triangle  $abc = \frac{1}{2}ABC$ , by *Corol. 6*, Theorem 7. In like manner, if you have a Triangle, which shall be the third Part of  $ABC$ , make it upon a Line  $= \frac{1}{3}BC$ , and within the same Parallels.

**Fig. 143.** *Case 3.* To make a Parallelogram  $ABCD$  twice (or thrice, &c.) as big as a given Parallelogram  $abcd$ . Draw  $CD = 2cd$ , and thereon make a Parallelogram  $ABCD$  within the same Parallels  $aB$  and  $dC$ , and the given Parallelogram lies within. *Corol. 5*, Theorem 7, the Parallelogram  $ABCD = 2abcd$ . In like manner, if you have a Parallelogram thrice as big as a given Parallelogram  $abcd$ , draw for its Base a Line  $= 3cd$ , and thereon Work as afore. But if the Parallelogram given be a Square, and you would have a Parallelogram which shall not be only double (or triple, &c.) to the given One, but a Square too; then Work by the following Method, which is universal.



se 4. To increase any plain Figure *Fig. 144.*  
 ever in a Proportion: given. For  
 nce: It is requir'd to make a Square  
 D, *Fig. 142*, which shall be twice  
 thrice, &c.) as big as a given Square  
*Fig. 143.* Draw the Line ED, and  
 on set off  $EB = 2ab$ , and  $BG = ab$ .  
 n find AB the mean Proportional be-  
 n BG (or  $ab$ .) and EF (or  $2ab$ .) A  
 re made upon AB, (*viz.* ABCD,  
*144.*) will be the Square requir'd.  
 by Theorem 10,  $ABCD : abcd ::$   
 $abq$ . But, because  $ab : AB :: AB :$   
 by Construction, therefore  $ABq =$   
 $2ab \Rightarrow 2abq$ ; In like manner, if a  
 re ABCD, *Fig. 145*, be requir'd, *Fig. 145.*  
 h shall be the Triple of the Square  
*Fig. 143*, the Square ABCD, made  
 AB the mean Proportional between  
 and  $3ab$ , will be the Square requir'd.  
 by Theorem 10, (as afore)  $ABCD :$   
 $abq$ . But, because by Con-  
 struction  $ab : AB :: AB : 3ab$ , therefore  
 $ABq = (ab \times 3ab \Rightarrow) 3abq$ . And so of  
 other Proportion, and of any other *Fig. 146,*  
 e whatever. For Instance, the Cir- & 147,  
 CABDE, (*Fig. 147.*) drawn with & 148.  
 Diameter BE, (or Semi-diameter  $ab$   
 e,) the mean Proportional between  
 the Diameter of the Circle  $(cabde)$  and  
 is the Double of the Circle  $(cabde)$ ,  
*146.* For, by Theorem 10,  $CABDE :$   
 $cabde ::$

$cabde : BEq : ebq$ . But, because by Construction  $eb : BE :: BE : 2eb$ , therefore  $BEq = 2ebq$ , and consequently the Circle  $CABDE = 2cabde$ . By Parity of Reasoning the Circle  $CABDE$ , (Fig. 148,)  $= 2cabde$ . Fig. 146.

Case 3. If it be requir'd to encrease a given Square  $abcd$ , (Fig. 143,) in a geometrical Proportion, *i. e.* if it be requir'd to make a Square, which shall be twice, or (twice, twice, *i. e.*) four Times, or (twice Four, *i. e.*) eight Times, as big as the given Square  $abcd$ , there is a shorter Way of Working. Namely, a Square made upon the Diagonal  $da$  of the given Square  $abcd$ , (or upon a Line equal to the said Diagonal) will be the Double of the given Square. For, by Theorem 12,  $daq = acq = 2abcd$ , by Defin. 41. Thus, the Square  $ABCD$ , (Fig. 144,) which was  $= 2abcd$  by the other Method, will likewise be found  $= 2abcd$  by this Method. For any Side  $AB$  of the Square  $ABCD$  will upon Trial by the Compasses be found just equal to the Diagonal of the Square  $abcd$ . In like manner a Square made upon the Diagonal of the Square  $ABCD$ , (Fig. 144,) will be the Quadruple of, (or four Times as big as) the given Square  $abcd$ . For, by

12,  $ADq = ACq + DCq = 2ABCD$ ,  
 Defn. 41. But  $ABCD = 2abcd$ , as a-  
 is shewn; and therefore  $2ABCD =$   
 4, by Axiom 10. Wherefore  $ADq =$   
 $CD = 4abcd$ , by Axiom 3.

6. And after the like manner may  
 be increased in a Geometrical  
 Portion. Namely, the Circle  $CABDE$ ,  
 147, describ'd with a Ray  $CB = ab$   
 the Circle  $cabde$ , Fig. 146, will be  
 as big as the Circle  $cabde$ . (And  
 the Circle  $CABDE$ , Fig. 148, de-  
 scrib'd with a Ray  $CB = AB$ , Fig. 147,  
 will be four Times as big as the Circle

For, by Theorem 12,  $baq =$   
 $caq$ ; and likewise  $BAq = BCq + CAq$ .  
 Therefore, and by Theorem 10, the Cir-  
 $CABDE : cabde :: BEq : ebq :: (\frac{1}{2}BEq)$   
 $(\frac{1}{2}ebq) : bcq$ . But, because by Con-  
 struction  $BC = ba$ , therefore  $BCq = baq$ .  
 Because  $baq = (bcq + caq) = 2bcq$ ;  
 therefore  $BCq = (baq) = 2bcq$ , and con-  
 sequently  $CABDE = 2cabde$ .

7. To diminish or lessen a Paral-  
 lelogram in a given Proportion. For In-  
 stance, to make a Parallelogram  $abcd$ ,  
 43, which shall be Half of a gi-  
 ven Parallelogram  $ABCD$  in the same  
 43. Take  $cd = \frac{1}{2}CD$ , and thereon  
 draw the Parallelogram  $abcd$ , lying within  
 the Parallels  $aB$  and  $cD$ , as  $ABCD$   
 lies

lies within. The Parallelogram  $ABCD$ , by *Corol. 5, Theorem 7.* If the Parallelogram found be not a Square, you would have it a Square, then by *Case 7, Problem 10.*

*Case 8.* To encrease or diminish other Figure, viz. a Trapezium or Polygon, &c. Divide it into Triangles, and by *Case 1, and 2,* encrease or diminish, as is requir'd, the several Triangles; and then make by *Problem 5,* a Figure similar to the given figure.



CH A P. III.

Containing the more useful and easy  
Elements of Geometry, which re-  
late to Solids or Bodies.

DEFINITIONS.

**Solid** is that, which has all the *Def. 1.*  
three Dimensions of Quantity, viz. *A Solid or*  
Length, Breadth, and Depth. And, be- *Body,*  
cause all Bodies have the said three Di- *what.*  
mensions, and they are found in Nothing  
but Bodies, therefore Solids are o-  
therwise call'd *Bodies.*

Every Solid is terminated or bounded *Def. 2.*  
by one or more Surfaces. *The Terms*  
*of Solids,*  
*what.*

**Similar Solids** are such, as are termi-  
nated by an equal Number of similar *Def. 3.*  
Surfaces; as two Dice. *Similar So-*  
*lids, what.*

**Solid Angle** is made by more than *Def. 4.*  
three Lines, which, being not in the same  
Plane, meet in some one Point; and  
consequently, a solid Angle always con-  
sists of more than two plain Angles;  
The Quantity of the said plain An-  
gles,

gles, is always less (\*) than four Right plain Angles : Or, in short, a solid Angle is the Meeting of three or more Planes in one Point.

**Def. 5.**

Regular  
Solids or  
Bodies,  
what; and  
how many.

A regular Solid or Body is, that which has all the Angles, Sides, and Planes that compose its Surface alike and equal. Whereas, then, to the making of a solid Angle, there are requir'd (by Definition) at least three plain Angles, and these together must be less than four Right plain Angles; and whereas (it has been above shewn, that) six Angles of an equilateral Triangle, four Angles of a Square, and three of an Hexagon, are equal to four Right plain Angles; and four Angles of a Pentagon, and three of an Hexagon, and all other following Polygons exceed four Right plain Angles; it remains for a solid Angle to be capable of being made only by the Meeting of three together, either of three, or of five equilateral Triangles, or of three Squares, or lastly of three Pentagons. Whence it follows, that there can be but the Five regular Bodies, described in the five next following Definitions.

(\*) See this prov'd, Prop. 21. Book II. of Euclid's Elements.

(†) In the Use of Theorem 1, and 3.

(II) *Tetraëdron* is a regular Body, bounded by four equal and equilateral angles.

Def. 6.

A Tetraëdron, what.

An *Hexaëdron*, commonly call'd a *Cube*, is a regular Body bounded by six equal faces. Such is a common Dy.

Def. 7.

A Cube, what.

An *Octaëdron* is a regular Body, bounded by eight equal and equilateral Triangles.

Def. 8.

An Octaëdron, what.

A *Dodecaëdron* is a regular Body, bounded by twelve regular Pentagons.

Def. 9.

A Dodecaëdron, what.

An *Icosaëdron* is a regular Body, bounded by twenty equal and equilateral Triangles.

Def. 10.

An Icosaëdron, what.

These are the five regular Bodies, otherwise known by the general Name of five *Platonick* Bodies; because (\*) Affections were much contemplated and studied by *Plato*, or at least his Followers the *Platonists*.

Def. 11.

The *Platonick* Bodies, what.

These and the Rest take their Names from Greek denoting their Number of Sides.

We learn from *Proclus*, that the celebrated *Euclid* was a *Platonist*, drew up his *Elements* to this End, might shew and illustrate the Affections of the regular Bodies. They are not so easily to be represented by being drawn on Paper, and therefore Figures are omitted. To give the young Student a tolerable Idea of them, it is requisite to make Wood or Pastboard, or the like.

Def. 12.

A Sphere  
or Globe,  
and its  
Axis and  
Rays,  
what.

Fig. 149.

Among other Solids or Bodies, which are not esteemed regular, the chief of these, which are describ'd in this and five Definitions next following. To begin with the Definition of a (+) Sphere or Globe, CABDE Fig. 149. It is a Solid, which may be conceiv'd to be generated or made by turning a Semicircle CABE (or CABD) round upon its Diameter AB kept fixt or unmov'd, where the Diameter is therefore call'd the Axis of the Sphere. Hence C the Center of the Semicircle, is also the Center of the Sphere; and all right Lines drawn from C the Center to the Surface AB of the Sphere, and call'd its Rays, are equal.

Def. 13.

A Cylinder,  
its  
Axis and  
Bases,  
what.

Fig. 150.

A (||) Cylinder is a Solid, which may be conceiv'd to be generated by turning a Parallelogram ABCD about upon one of its Sides.

(+) As a Circle may be justly esteem'd in regular Figures, so may a Sphere of Solids. The Reason, why they are not comprehended under the Name of regular Figures, seems to be this, viz. they are (as it were) the common Measure of all regular Figures (which are distinguish'd by the Name of) regular Figures, as all regular Planes may be inscrib'd in a Circle, and all regular Bodies be inscrib'd in a Sphere.

(||) A Cylinder is a Greek Word, denoting a Roller, such as a Garden-roller; and a Cone denotes in Greek a Cone.



Sides AB kept fixt. Which Side is before call'd the *Axis* of the Cylinder; the *Bases* of the Cylinder are the Circles ACGH and BDEF, describ'd the two opposite Sides AC and BD.

(II) Cone is a Solid Figure, which be conceiv'd to be generated by turning a Triangle ABC about upon one Side AB kept fixt. Which Leg is there call'd the *Axis* of the Cone; and the *Base* of the Cone is the Circle BCDE,

Def. 14.

A Cone, its Axis and Bases, what.

Fig. 152.

T 3

made

any Thing that is shap'd like a *Sugar-loaf*. And the said Words are taken to denote all Solids of the Figures or Shapes. And it is observable, that is evidently a Part of a Cylinder. It is also to be noted, that the Interfection of any Plane, with the Cone, is call'd a *conick Section*, or in one Coni-section, and is distinguish'd into three Species, a *Parabola*, an *Ellipsis*, and an *Hyperbola*, being of considerable Use in *Mathematicks*, (so as whole Treatises writ concerning them,) and being used in the young Gentleman's *Astronomy*, I have added some Draughts to represent them, (as may be on a Plane,) viz. the three Figures, numbered 165, 166, 167, &c. In which Figures, ABC is the Cone; DE (Fig. 165.) represents a *Parabola* (Fig. 166.) an *Ellipsis*, and HI (Fig. 167.) an *Hyperbola*. Whence it appears, that the Difference between the said Coni-sections, is in general this: A *Parabola* is the Section made, when the Cone ABC is cut by a Plane, DE parallel to the Side AB: An *Hyperbola* is the Section made, when the Cone ABC is cut by a Plane, HI, so as to meet the Side AB, if extended: An *Ellipsis* is the Section made, when the Cone ABC

made by the moving round of the  
L<sup>g</sup> AC.

Def. 15.

A Prism,  
what.

Fig. 154.

A *Prism* is a solid Figure, termin-  
ed by any Number of Planes, whereof  
two opposite ADE and BCF (which  
be esteem'd the top and bottom Plane  
of the Prism) are equal, and similar,  
parallel; and all the other Planes BC  
BFEG, &c. are Parallelograms.  
Several Sorts of Prisms take the Name  
of *Triangular*, *Quadrangular*, &c. from  
the Figure of their Bases,  
&c.

Def. 16.

A Parallelepiped,  
what.

A *Parallelepiped* is (one Sort of  
being no other than) a solid Figure  
terminated by six Parallelograms, after  
a manner as that every two opposite  
of it are both parallel and equal  
to another. Hence it is to be noted  
that an Hexaedron or Cube is no other  
than one Sort of Parallelepiped, viz.  
which has all its Sides equal and square.

Def. 17.

A Pyramid,  
what.

Fig. 156.

A *Pyramid* is a solid Figure, ter-  
minated by any Number of Planes,  
from one plain Basis BCDE grow-

---

ABC is cut by a Plane FG obliquely, so as to  
Sides AB and BC. Hence an Ellipsis, encom-  
passing the Surface of the Cone, includes a Space, as is re-  
Fig. 168. The other two are continued in insid-  
the Cone, and so include not a Space, as Fig. 16

er and narrower, till they end in a  
at A.

By a *right* or *rectangular* Cylinder, (or  
e, or Prism, or Pyramid,) is to be  
understood a Cylinder, (or Cone, &c.)  
whose Axis AB is perpendicular to its  
BDEF, as Fig. 150, and also Fig.  
154, and 156. By an *oblique* or *ob-*  
*angular* Cylinder, (or Cone, &c.) is  
understood a Cylinder, (or Cone,  
whose Axis AB falls obliquely on  
BDEF, as Fig. 151, and also  
153, 155, 157. And it is to be  
observ'd, that the Altitude or Height  
of an oblique Cylinder or Cone, &c. is  
measur'd by the Axis of a right  
Cylinder or Cone, &c. within the same  
parallel Planes. Thus, AB, Fig. 150, is  
the Height of the oblique Cylinder, Fig.  
151. And AB, Fig. 152, is the Height  
of the oblique Cone, Fig. 153.

Def. 18.

The Altitude of a  
Cylinder,  
or Cone,  
&c. what.

multiply a Surface by (or into) a  
Line, is, by drawing the Surface perpen-  
dicular along the Line; to describe a

Def. 19.

To multi-  
ply a Sur-  
face by a  
Line,  
what.

Thus, the Square ABCD, (Fig.  
159) being multiplied into the right  
Line EF, (Fig. 160,) will describe a  
Cylinder. And the Oblong ABCD, Fig.  
161, being multiplied into the Line EF,  
(Fig. 164,) will describe an ob-  
lique Parallelepiped. In like manner,

the Triangle ABC, Fig. 161, being multiplied into the Line EF or KL, will describe a triangular Prism; as the Trapezium ABCD, Fig. 162, will a quadrangular Prism; and the Pentagon ABCDE, Fig. 163, will a quinquangular Prism and so of other Polygons. Lastly the Circle CABDE, Fig. 146, multiplied to EF or KL, will describe a Cylinder. Hence it is evident, that EF or KL may be look'd on as (the Measure of) the Height of the aforesaid Solids, or as the Height of several Planes, multiplied by EF or KL may be look'd on as the Basis of the said Solids. And from this Method of generating the afore-mention'd Solids, most naturally deduced the Method of finding the Solidity (or solid Content) of the said several Bodies, as also of

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(\*) It is to be observ'd, that, because there is more than one Way for conceiving Solids to be generated, therefore it is said above, Def. 13, 14, *Or* the Cylinder, Cone, &c. may be conceiv'd to be generated as is there describ'd. And it is also to be observ'd, that the Line EF or KL, into which the Surface is multiplied according to the Method describ'd, Def. 13, 14, is answer'd to the Axis of a Cylinder or Cone, &c. by which the Height of the said Solids are said to be measured, Defn. 18; that is, the said Line EF or KL, is equal to the Axis of a right Cylinder or Cone, &c. generated by multiplying a Circle into EF or KL.



lies terminated by plain Surfaces. The  
 nsuration of which shall be compris'd  
 he four following Theorems and their  
 ollaries.

## THEOREM I.

The Solidity of a Cube, or (†) other  
 allelepipied, or any (other) Prism,  
 also of a Cylinder, is found by multi-  
 ing its Basis into its Height.

For this is (as appears from *Defin. 19.*)  
 other than the very Way, whereby  
 said several solid Figures are describ'd  
 roduc'd; and, consequently, hereby  
 solid Content must be discover'd.

Illustrate the Matter by an Instance  
 two. By the Motion of the Square  
 CD, (*Fig. 158.*) perpendicularly a-  
 the Line EF, *Fig. 160.* there is pro-  
 d according to *Defin. 19.* a Cube;  
 Line EF being equal to a Side AB of  
 Square ABCD. Let therefore AB be  
 os'd three Foot long, and conse-  
 tly (by *Defin. 41, Chap. 1.*) the A-  
 of the Square, ( $3 \times 3$ , i. e.) nine Feet  
 re. Wherefore, the Square ABCD

---

For a Cube is one Sort of Parallelepiped, and a  
 pipied is one Sort of Prism.

being

being moved upon the Line EF, from E to G, *i. e.* one Foot; it is evident, it will describe by such its Motion nine solid Feet. And moving from G to H, *i. e.* one Foot further, it will describe nine solid Feet more. And lastly, by its Motion from H to F, *i. e.* a third Foot, it will likewise describe nine solid Feet more. So that the Square ABCD by its Motion from E to F, will describe the nine solid Feet, *i. e.* 27, which is the Product of the Basis or Square AB = nine Feet square, multiplied into the Height = three Foot long, *viz.*  $9 \text{ sq.} \times 3 \text{ f.} = 27 \text{ f. cubical}$ , or 27 Solid Feet, which are each a Foot in Length, Breadth, and Depth.

Likewise, by *Defn.* 19, the Triangle ABC, *Fig.* 161, moving perpendicularly along KL, (suppos'd equal to six Feet in Length,) describes a triangular Prism. Wherefore, supposing the Side AB of the Triangle to be three Foot, and its Height to be four Foot, its Area will be six Feet square. Wherefore, the said Triangle moving upon KL, from K to L, will describe six solid Feet; and moving from O to M, will describe six solid Feet more; and moving from M to P, will describe six solid Feet more. And moving from K to L, it will describe

... six solid Feet, i. e. 36; which is  
Product of the Basis (or Triangle),  
 $= 6 \text{ f. sq.}$  multiplied into KL, the  
Height (of the said triangular Prism)  
 $= 6 \text{ f.}$  For  $6 \text{ f. sq.} \times 6 \text{ f.} = 36 \text{ f.}$  solid  
cubical.

In like manner, supposing the Ray of  
Circle to be three Foot long, and  
Semicircumference to be nine, the Area  
of the said Circle will be twenty-seven  
square. Wherefore, the Solidity of  
Cylinder, describ'd by the moving of  
Circle perpendicularly along EF =  
three Foot long, will be  $27 \times 3 = 81 \text{ f.}$   
For it is evident, that the said  
Circle moving upon EF, from E to G,  
describe twenty-seven Feet solid;  
moving from G to H, will describe  
twenty-seven Feet solid more; and, last-  
moving from A to F, will describe  
twenty-seven more solid Feet. And so  
moving from E to F, will describe a  
Cylinder, whose Solidity will be two  
hundred and forty-three solid Feet. And  
of the rest.

Now, as in Surfaces it has been shewn,  
obliquangular Parallelograms are e-  
quivalent to rectangular Parallelograms, upon  
the same or equal Bases, and (within the  
same Parallels, or) of the same Altitude;  
by Parity of Reason, obliquangular  
Prisms

Prisms and Parallelepipeds are equal to rectangular Ones upon the same equal Bases, and of the same Altitude. And therefore, the Solidity of oblique angular Parallelepipeds or other Prisms is found out by the same Method, as the Solidity of rectangular Parallelepipeds or other Prisms, viz. by this Theorem 1. And the same holds good, by a Parity of Reason, between rectangular and obliquangular Cyllinders, upon the same or equal Bases, and of the same Altitude.

## T H E O R E M II.

The Solidity of a Pyramid, or Cone, is found by multiplying either the whole Basis into a third Part of its Height, or its whole Height into a third Part of its Basis, or (which still comes to the same) by multiplying the whole Basis into the whole Height, and dividing the Product by three.

For it is (II) demonstrable, that every Pyramid is a third Part of a Prism upon the same or an equal Basis, and of the same Altitude; and likewise, that every

---

(II) See Euclid's Elements, B. 12. Prop. 7, and 10.



is the third Part of a Cylinder upon the same or an equal Basis, and of same Altitude. Wherefore, since by prem 1, every Prism and Cone =  $\frac{1}{3}$  Altitude, it follows, that a Pyram ( $= \frac{1}{3}$  Prism) = Base  $\times \frac{1}{3}$  Altitude, Base  $\times$  Altitude, and likewise a Cone (Cylinder) = Base  $\times \frac{1}{3}$  Altitude, or  $\frac{1}{3}$  Base  $\times$  Altitude, &c.

Prop. 1. As a Tetraedron is no other Sort of Pyramid, and therefore its Solidity is to be found by this Theorem, so an Octaedron, being no other than two Tetraedrons or Pyramids together at their Bases, hence the Content of the common Basis being found, and multiplied by a third Part of the Height of the Octaedron, (which is the Sum of the two Heights of the two Tetraedrons or Pyramids, into which the Octaedron is divisible,) will give the Solidity of the Octaedron.

To measure an Octaedron.

Prop. 2. Forasmuch as a Dodecaedron may be conceiv'd to be made up of 12 Pyramids, whose several quadrilateral Bases are all equal one to the other, and make the twelve several Surfaces of the Dodecaedron, and whose Vertex's consequently meet together in the Center of the said Solid, having found the Area of one quadrilateral

To measure a Dodecaedron.

quingangular Base, and multipli-  
into one Third or Half the Height of  
Solid, you will have the Solidity of  
Pyramid. And this being multipli-  
12, will give the Solidity of the  
Dodecaedron.

To measure  
an Icosae-  
dron.

*Corol. 3.* The Icosaedron being  
up of twenty equal Pyramids, w  
Bases are so many equal Triangles  
Surface of the Icosaedron; and w  
Vertex's consequently meet in the C  
of the said Body; it follows, the  
Area of one of the twenty Triangle  
ing found, and multiplied by a  
Part of (the Height of one Pyra  
i. e.) Half the Height of the Icosae  
the Product will give the Solidity of  
Pyramid, or 20th Part of the said  
faedron. And, consequently, this  
duct being multiplied again by 20,  
give the Solidity of the whole I  
dron. And thus there has been  
the Method of Measuring, as of  
Solids, so of the five Regular (or  
nick) Bodies. For the Hexaedro  
Cube is one Sort of Parallelepiped  
so its Mensuration is taught in  
rem 1. And the Tetraedron is on  
of Pyramid, and so its Mensurati  
taught in Theorem 2. And the M  
ration of the other three regular B

ought in the other three Corollaries  
 rem 2.

# THEOREM III.

Parallelepipeds and (other) Prisms,  
 also Cylinders, if they have the  
 or equal Bases, are one to the o-  
 as their Altitudes; or if they have  
 same or equal Altitude, are one to  
 ther as their Bases.

This might be accounted for, as to  
 elepipeds and other Prisms, from  
 Relation they bear to Parallelo-  
 and Triangles, between which  
 is the like Proportion, (as has  
 shewn, Chap. 1, Theorem 7.) But  
 I rather account for it from the  
 er of producing Parallelepipeds  
 prisms, according to *Defn. 19*, of  
 Chap. 3; because, hereby the like  
 tion between Cylinders will also  
 counted for. According then to  
 19, a Parallelepiped is produ-  
 multiplying a Parallelogram in-  
 ight Line, which Parallelogram  
 be conceiv'd to be the Basis of  
 parallelepiped, and the right Line  
 Altitude. Wherefore Parallelepi-  
 which have the same or equal  
 are (or may be conceiv'd to be)  
 produced,

produced by the same or equal Parallelograms. And therefore, if the same or equal Parallelograms be multiplied into the same or equal Lines, it is evident, that the Parallelepiped thus produced (must) be equal. But if the same or equal Parallelograms be multiplied into Lines of different Length, suppose one twice as long as the other, then it is evident, that the Parallelepiped produced by the Multiplication of a Parallelogram upon a longer Line, must be twice as big as the Parallelepiped produced by the Multiplication of the same, or an equal Parallelogram into the shorter Line. So of any other Proportion. To illustrate the Matter in Numbers: Suppose the Parallelograms to be six Foot square, and the Lines each two Foot long. It is evident, that the Parallelepipeds, produced by the Multiplication of the said Parallelograms into the said Lines, will be twelve Foot cubical, and therefore equal. But, suppose one of the equal Parallelograms to be multiplied into a Line two Foot long, and the other equal Parallelogram be multiplied into a Line twice as long, *viz.* four Foot long; it is evident, that the



ipped, produced by the former multiplication, will contain twelve cubical Feet; and the other Parallelepiped, produced by the latter Multiplication, will contain twenty-four Feet cubical; and consequently, the Parallelepipeds will be one to the other, as (the Lines by which the Parallelograms were multiplied, *i. e.*) their Altitudes. In like Manner, suppose one Parallelogram to contain six Feet square, the other three, and both to be multiplied into a Line four Foot long; it is evident, that the Parallelepiped, produced by the former Multiplication, will contain twenty-four cubical Feet; the Parallelepiped, produced by the latter Multiplication, will contain twelve cubical Feet; and consequently, Parallelepipeds, having the same equal Altitude, will be one to the other, as are (the Parallelograms multiplied, *i. e.*) their Bases. And the like may be illustrated after the same Manner to all Prisms, and also Cylinders. From whence the Truth of this Proposition evidently appears.

Pyramids, and also Cones, if they have the same or equal Bases, are to one another as their Altitudes; and if they have the same or equal Altitudes,

U

titudes,

titudes, they are one to the other as their Bases. For every Pyramid is the third Part of a Prism, and every Cone is a third Part of a Cylinder upon the same or equal Bases, and the same or equal Altitude. Wherefore, by *Axiom 13*, Pyramids will be in the same Proportion one to the other as the Prisms whereof they are third Parts; and Cones will have the same Proportion one to the other, as the Cylinders whereof they are third Parts.

#### THEOREM IV.

Similar Solids are one to the other as the Cubes of their homologous Sides.

This likewise may be clearly and universally accounted for from *Definition 10* of this *Chap. 3*, and *Theorem 10*, *Prop. 1*. For similar Planes being one to the other, as the Squares of their homologous Sides, and similar Solids being produced by multiplying similar Planes into the homologous Sides of the similar Solids, (by which Multiplication, the particular Squares, *viz.* Feet, or Inches, &c. contain'd in the similar Planes, describe so many particular Cubes, *i. e.* cubical Feet or

etc. in the similar Solids,) hence follows, that similar Solids are one to the other, as the Cubes of their homologous Sides. Where it is to be noted, that the Diameters of Cylinders, and consequently (of the Bases) Cones, are the homologous Sides of Cylinders and Cones: Whence similar Cylinders or Cones are one to the other, as the Cubes of their Diame-

ters. And, because as all Circles may be considered on as similar Planes, so all Spheres generated by the Circumvolution of Circles may by Parity of Reason be considered on also as similar Solids; therefore Spheres (as well as other similar Solids) are one to the other, as the Cubes of their Diameters. For Instance, suppose the Diameter of one Sphere or Globe to be two Inches, and the Diameter of another to be four; this latter Globe will be to the former, as  $(4 \times 4 \times 4, i. e.) 64$  is to  $(2 \times 2 \times 2, i. e.) 8$ , that is, it will be eight Times bigger. For  $8) 64 (8$ .

Prop. Hence is deducible a Way of Increasing or Diminishing a Solid. For Instance: I have a Globe whose Diameter is two Inches; and I would have a Globe which shall be eight Times

titudes, they are one to the other as their Bases. For every Pyramid is the third Part of a Prism, and every Cone is a third Part of a Cylinder upon the same or equal Bases, and the same or equal Altitude. Wherefore, by *Axiom* 13, Pyramids will be in the same Proportion one to the other as the Prisms whereof they are the third Parts; and Cones will have the same Proportion one to the other, as the Cylinders whereof they are third Parts.

#### THEOREM IV.

Similar Solids are one to the other as the Cubes of their homologous Sides.

This likewise may be clearly and universally accounted for from *Definition* of this *Chap.* 3, and *Theorem* 10, 11. For similar Planes being one to the other, as the Squares of their homologous Sides, and similar Solids being produced by multiplying similar Planes into the homologous Sides of the similar Solids, (by which Multiplication, the particular Squares, *viz.* Feet, or Inches, &c. contain'd in the similar Planes, describe so many particular Cubes, *i. e.* cubical Feet or



, &c. in the similar Solids,) hence follows, that similar Solids are one to the other, as the Cubes of their homologous Sides. Where it is to be noted, that the Diameters of Cylinders, and consequently (of the Bases) Cones, are the homologous Sides of Cylinders and Cones: Whence similar Cylinders or Cones are one to the other, as the Cubes of their Diameters.

And, because as all Circles may be regarded on as similar Planes, so all Spheres generated by the Circumvolution of Circles may by Parity of Reason be regarded on also as similar Solids; therefore Spheres (as well as other similar Solids) are one to the other, as the Cubes of their Diameters. For Instance, suppose the Diameter of one Sphere or Globe to be two Inches, and the Diameter of another to be four; this latter Globe will be to the former, as  $(4 \times 4 \times 4, i. e.) 64$  is to  $(2 \times 2 \times 2, i. e.) 8$ , that is, it will be eight Times bigger. For  $8 \times 64 = 512$ .

Hence is deducible a Way of increasing or Diminishing a Solid. For Instance: I have a Globe whose Diameter is two Inches; and I would have a Globe which shall be eight Times

as big. Wherefore, I take for the  
 meter of the Globe requir'd such  
 Number of Inches, viz. 4, as be-  
 cubed, will be eight Times as big  
 the Cube of 2. Or, if the Diam-  
 of my Globe be 4 Inches, and I  
 have one eight Times as little,  
 I take for the Diameter of the G  
 requir'd such a Number, viz. 2, as  
 ing cubed, will be an eighth Part o  
 Cube of 4.

And thus I have gone through  
 Elements of Geometry, as are an-  
 nable to the Design of this Geomet-  
 Treatise.

---

**F I N I S.**

# A T A B L E

SHEWING,

what Propositions of *Euclid*, the Theorems and Problems of this Geometrical Treatise answer.

| Gen. Geomery.                         |             | Euclid. |
|---------------------------------------|-------------|---------|
| A P I.                                | Prop. Book. |         |
| THEOREM I.                            | 13          | I       |
| Theorem II.                           | 15          | I       |
| Prop. III, and }<br>Corollary I, 2, } | 29          | I       |
| Corollary 3.                          | 27, 28      | I       |
| Prop. IV.                             | 32          | I       |
| Prop. V.                              | 8           | I       |
| Corollary 1.                          | 4           | I       |
| Corollary 2.                          | 5           | I       |
| Corollary 5.                          | 34          | I       |
| Corollary 6.                          | 43          | I       |
| Prop. VI.                             | 20          | 3       |
| Corollary 1.                          | 21          | 3       |
| Corollary 2.                          | 31          | 3       |
| Corollary 3.                          | 22          | 3       |
| Prop. VII.                            | 35          | I       |
| Corollary 1.                          | 36          | I       |
| Corollary 2.                          | 37          | I       |
| Corollary 3.                          | 38          | I       |

*Young*

# A TABLE, &c.

Young Gent. Geometry.

## CHAP. I.

Prop. Bo

|                   |            |
|-------------------|------------|
| Corollary 4.      | 41         |
| Corollary 5, & 6. | 1          |
| Theorem VIII.     | 2          |
| Theorem IX.       | 4          |
| Theorem X.        | 19, 20     |
| Theorem XI.       | 8          |
| Theorem XII.      | { 47<br>31 |

## CHAP. II.

Prop. Bo

|            |            |
|------------|------------|
| Problem I. | 31         |
| II.        | 23         |
| III.       | 9          |
| IV.        | 22         |
| V.         | 10         |
| VI.        | 12         |
| VII.       | 13         |
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| IX.        | 46         |
| X.         | 42, 44, 45 |
| XI.        |            |
| XII.       |            |

## CHAP. III.

Prop. B

|            |                  |
|------------|------------------|
| Theorem I. |                  |
| II.        | 7, 10            |
| III.       | { 32<br>5, 6, 11 |
| IV.        | { 33<br>12, 18   |

ERR



# DATA to the Young Gentleman's Arithmetick, and Geometry.

| Line     | Instead of                                                                                                                                          | Read                                                      |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| penult.  | 296785431 ... 0.                                                                                                                                    | 1296785431 ... 1.                                         |
| ult.     | 6086417841 ... 0.                                                                                                                                   | 7086417841 ... 1.                                         |
| 5        | 1134587693.                                                                                                                                         | 1134587692.                                               |
| and 22.  | Chap. 5.                                                                                                                                            | Chap. 10.                                                 |
| 11.      | 6789.                                                                                                                                               | 6786.                                                     |
| 12.      | 2635964.                                                                                                                                            | 2635984.                                                  |
| 23.      | 3006.                                                                                                                                               | 3006.                                                     |
| 10.      | (i. e.) (.034.                                                                                                                                      | (i. e.) .034.                                             |
| 22.      | I, III, and IV.                                                                                                                                     | III and IV.                                               |
| 10.      | What follows here (at the End of Chap. 7.) concerning Sexagesimal Fractions, is placed through Mistake, pag. 73. §. 16. viz. at the End of Chap. 8. |                                                           |
| 8.       | 2 <sup>20</sup> and 6 <sup>12</sup> .                                                                                                               | 2 <sup>20</sup> of a Pound, and 6 <sup>12</sup> of a Sh.  |
| 7.       | 11.                                                                                                                                                 | 12.                                                       |
| 4.       | to its least Equivalent.                                                                                                                            | to a less Equivalent.                                     |
| 2 A+E    | Ac&c. (Aq+2AE+Eq) Ac&c. (A+E.                                                                                                                       | Aq+2AE+Eq) Ac&c. (A+E.                                    |
| 8.       | Sum of Common Stock, multiplied into Com-                                                                                                           | Sum arising from part. Shares, multiplied into particular |
|          | mon Gain.                                                                                                                                           | Times, and added together.                                |
| 28.      | $Z = a + w \times T.$                                                                                                                               | $Z = a + w \times T.$                                     |
| 8.       | 144) 2284.                                                                                                                                          | 114) 2284.                                                |
| 9.       | 1728 (12.                                                                                                                                           | 1738 (12.                                                 |
| 2        | 187149248 (572.                                                                                                                                     | 187149248 (572.                                           |
| 6        | 1.07918.                                                                                                                                            | 1.07918.                                                  |
| 17.      | Sector of a Circle.                                                                                                                                 | Sector of a Circle, as in Fig. 4 or 6.                    |
| 25.      | long Square.                                                                                                                                        | a long Square.                                            |
| 21.      | CA : AC :: ca : ab.                                                                                                                                 | CA : AB :: ca : ab.                                       |
| 20.      | Angle CBA.                                                                                                                                          | Angle BCA.                                                |
| 14.      | Angle ABC = CAB:                                                                                                                                    | Angle (BCA or) ACB = CAB.                                 |
| ult.     | opposite Angle B.                                                                                                                                   | opposite Angle A.                                         |
| 29.      | Semicircumference ABC.                                                                                                                              | Semicircumference AEC.                                    |
| 7.       | EH to FD.                                                                                                                                           | FH to ED.                                                 |
| 11.      | Proposition 5.                                                                                                                                      | Theorem 5.                                                |
| 6.       | AC or AG.                                                                                                                                           | AC or KG.                                                 |
| 10.      | BC = BD + BC.                                                                                                                                       | BC = BD = DC.                                             |
| ult.     | ABq + bCq.                                                                                                                                          | ABq + ACq.                                                |
| 15.      | Line BD.                                                                                                                                            | Line CD.                                                  |
| 11, 12.  | and acdb = ACDB.                                                                                                                                    | and cdb = CDB.                                            |
| 4, 5, 6. | 101, 102, 103.                                                                                                                                      | 104, 105, 106.                                            |
| 11.      | Length with the given Lines.                                                                                                                        | Length upon AD.                                           |
| 3        | FA = A.                                                                                                                                             | FH = A.                                                   |
| 3.       | FGIH = FGKL.                                                                                                                                        | FGIH + FGKL.                                              |
| ult.     | Parallelogram acgb.                                                                                                                                 | Parallelogram acgh.                                       |
| 10.      | and ABC.                                                                                                                                            | as ABC.                                                   |
| 4.       | Fig. 142.                                                                                                                                           | Fig. 144.                                                 |
| 28, 29.  | Semidiameter cb or ce.                                                                                                                              | Semidiameter CB or CE.                                    |
| 17.      | Depth, and Height.                                                                                                                                  | Depth or Height.                                          |
| 23, 24.  | two hundred and forty-three.                                                                                                                        | eighty-one.                                               |

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*Fig.*

A

*Fig.*

C



A

B

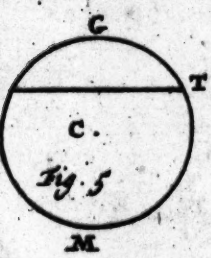
*Fig.*



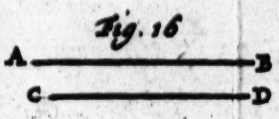
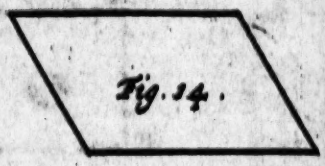
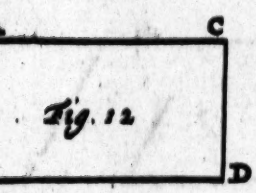
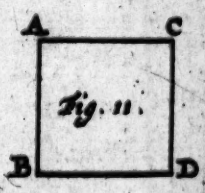
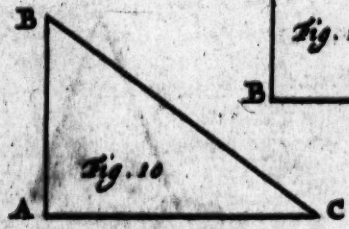
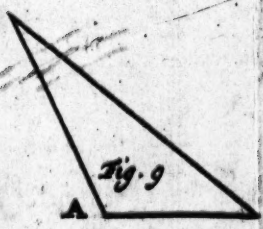
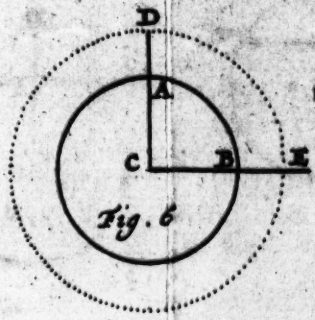
Fig. 1.



Fig. 2.



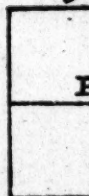
Geom. Plate 1. All figures to be drawn with  
latter end of a compass, and to fold over the point of  
the work.

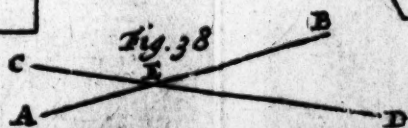
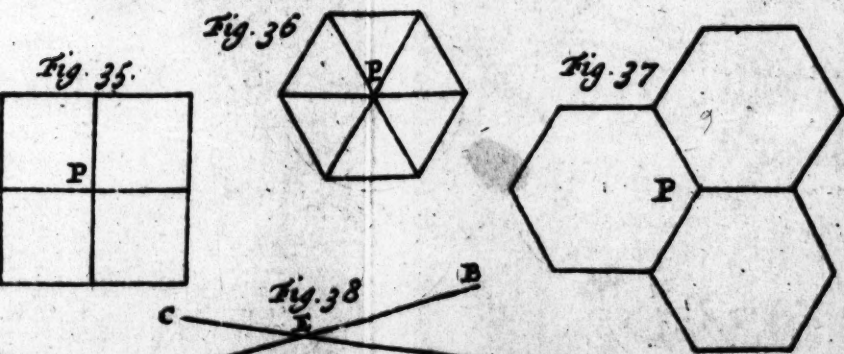
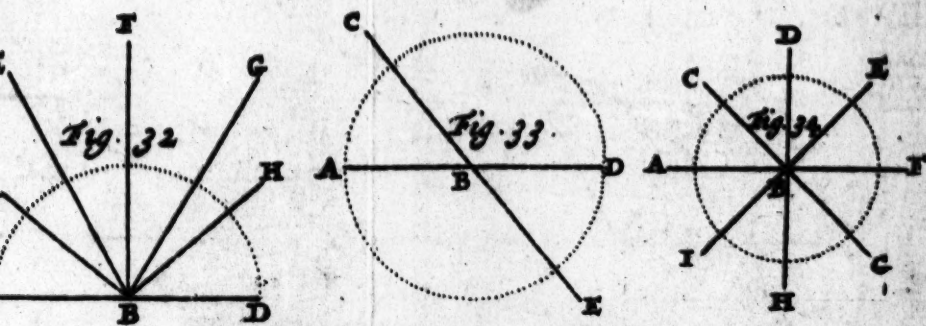
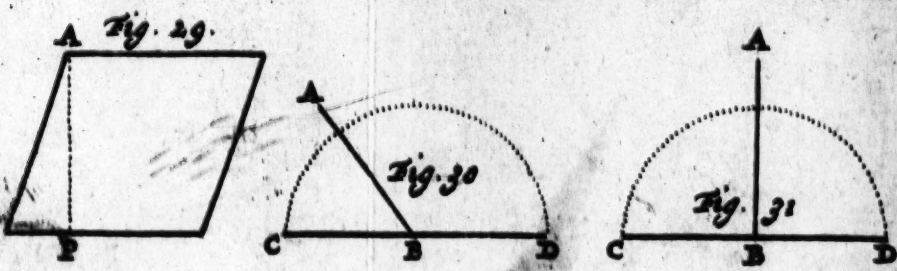
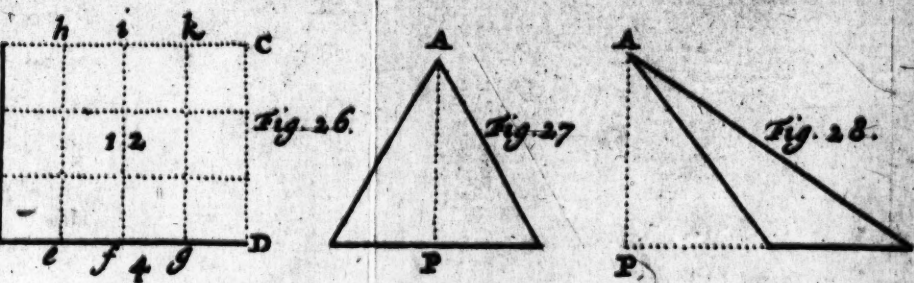
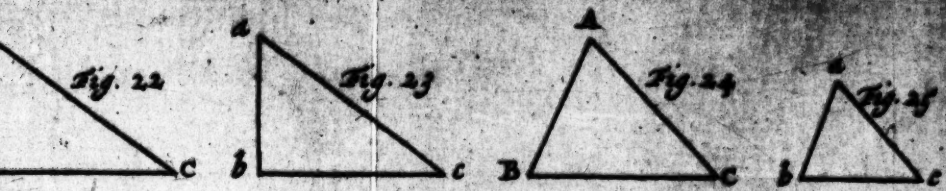






Fig





Height of  $\frac{1}{2}$   
Arch of  $\frac{1}{2}$

Arch

Q  
A





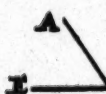
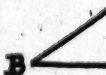


Fig. 47



Fig. 51

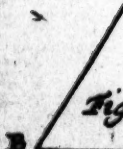


Fig. 52

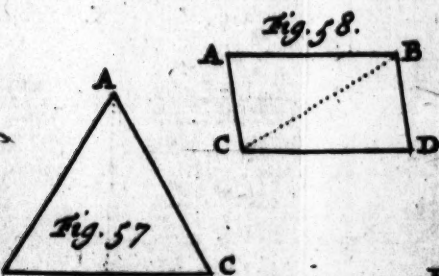
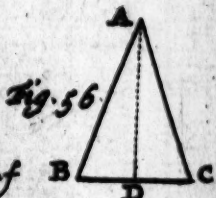
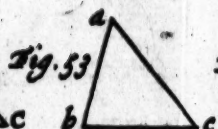
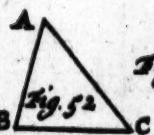
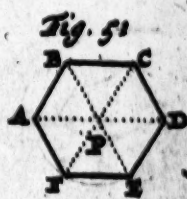
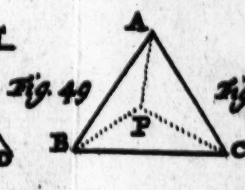
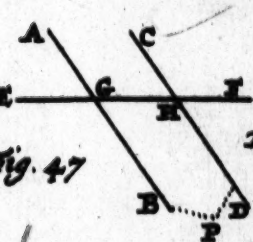
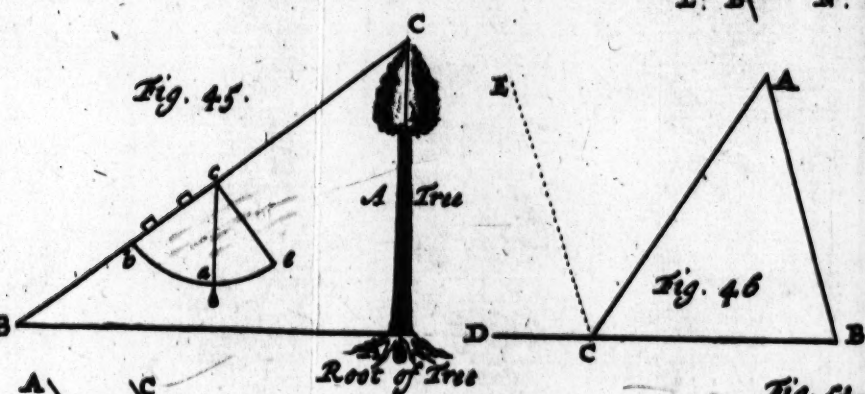
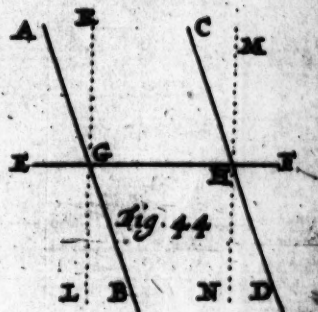
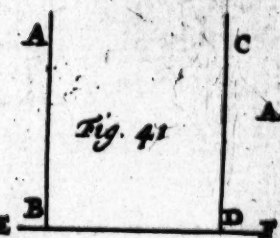


Fig. 58.

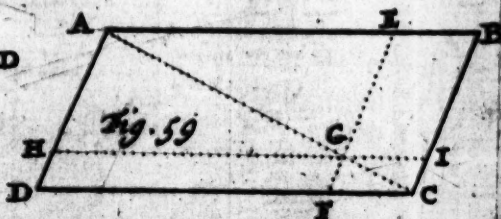


Fig. 80



E

Fig. 68



Fig. 60

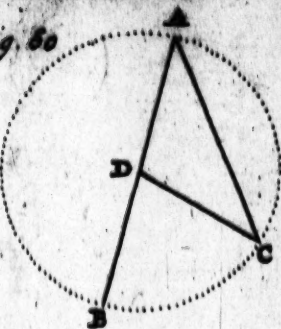


Fig. 61

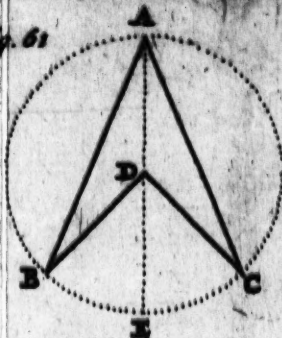


Fig. 62

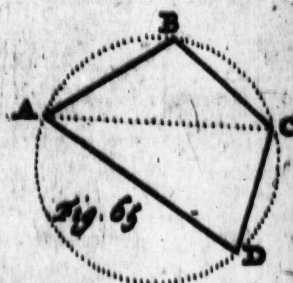
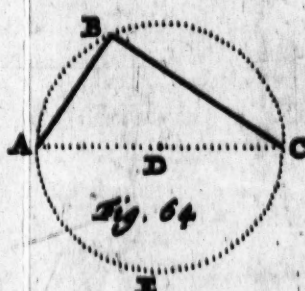
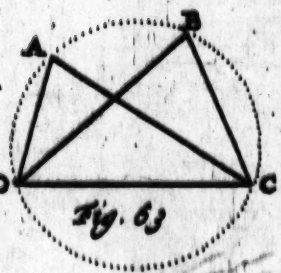
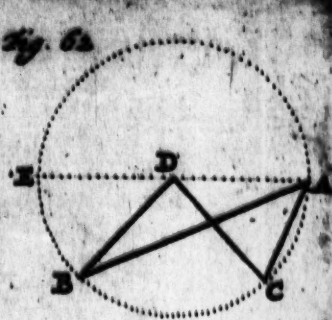


Fig. 66

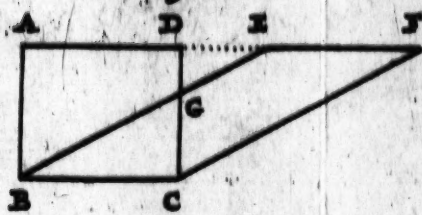


Fig. 67

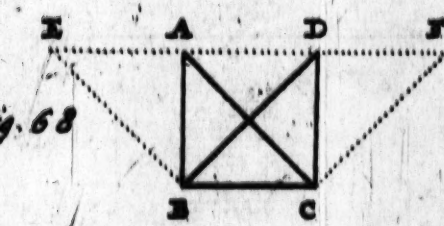


Fig. 69

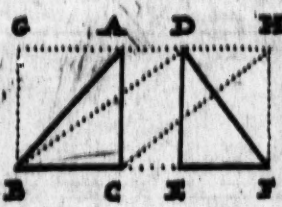


Fig. 70

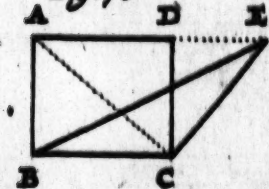


Fig. 71

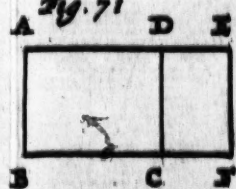
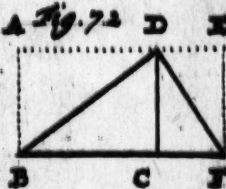
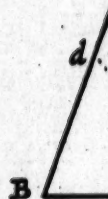
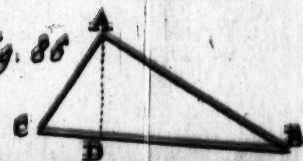
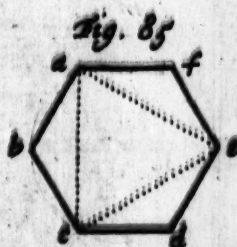
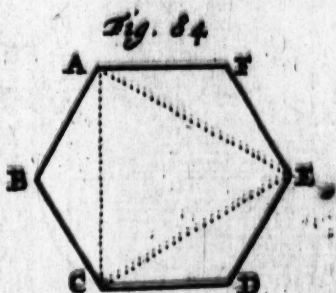
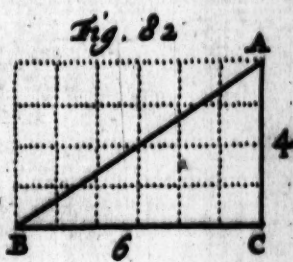
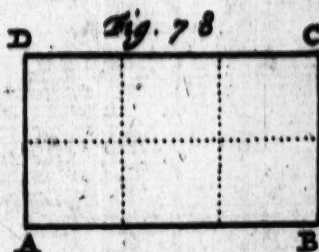
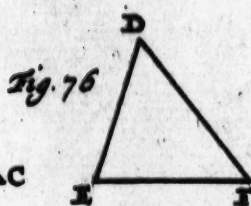
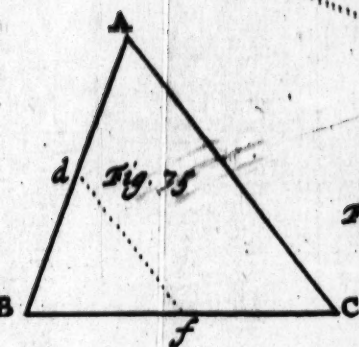
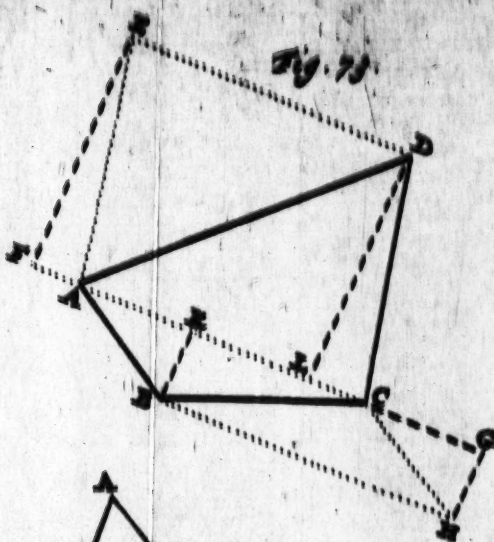


Fig. 72









Geom



Fig

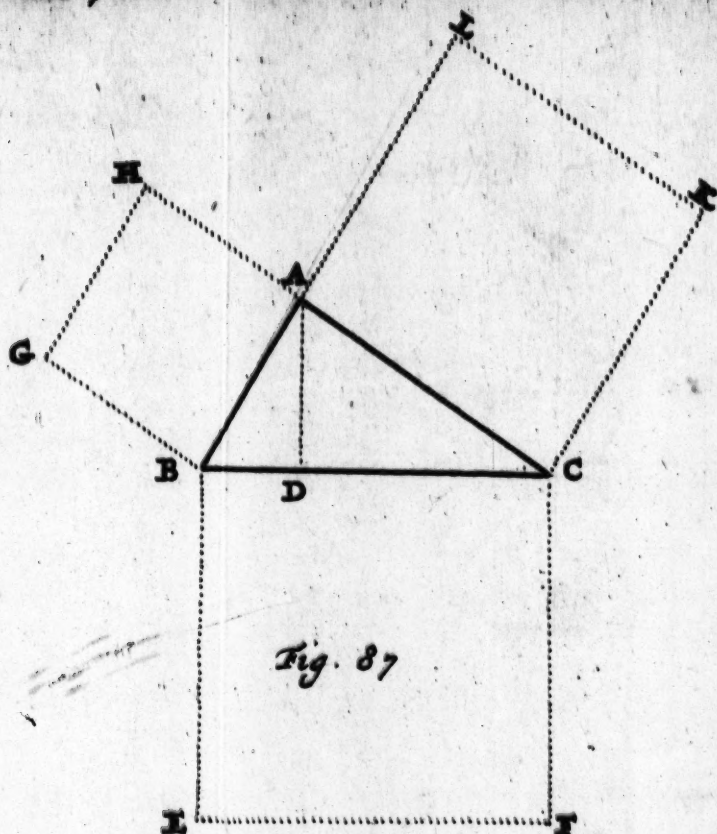


Fig. 87

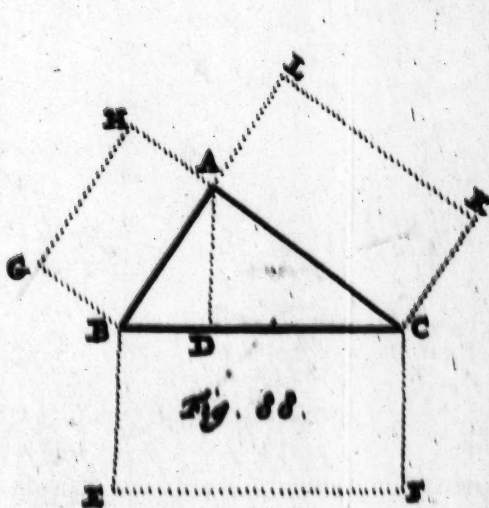


Fig. 88.



Fig. 89

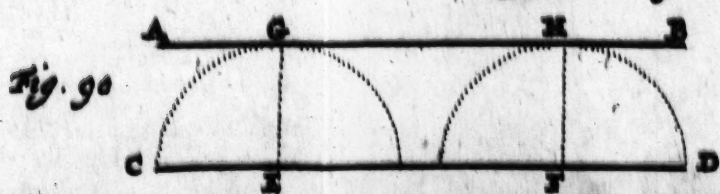


Fig. 90



A —  
C —

Fig. 5

D —

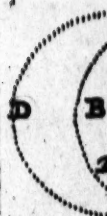


Fig.



Fig. 91

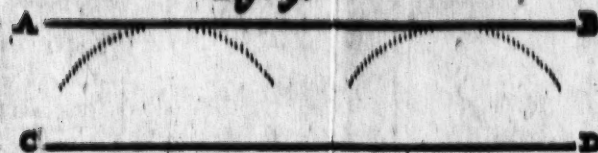


Fig. 92



Fig. 94

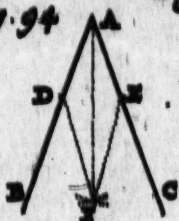


Fig. 95

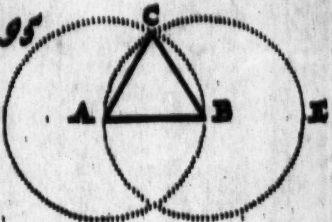


Fig. 93



Fig. 96

Fig. 97

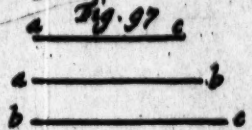


Fig. 97

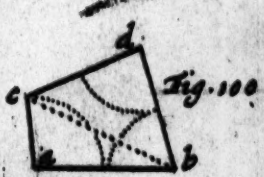
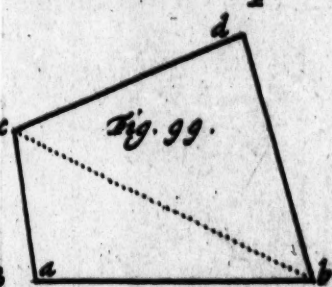
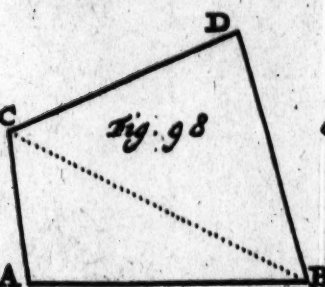
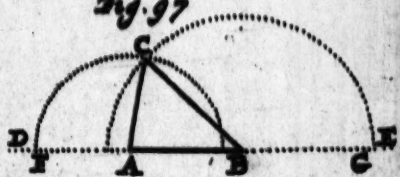


Fig. 102



Fig. 103

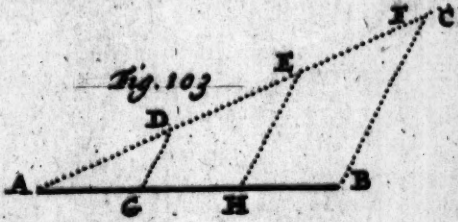


Fig. 104



Fig. 105

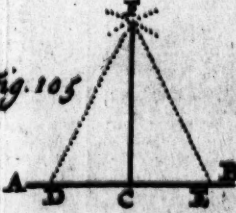


Fig. 106

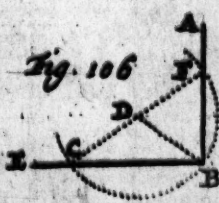


Fig. 107



Fig. 110



Fig. 112



Fig. 11

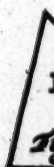


Fig.

Fig. 107

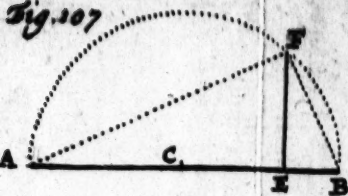


Fig. 108

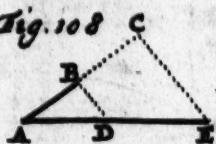


Fig. 109

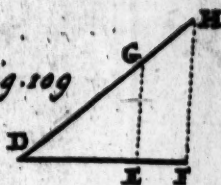


Fig. 110

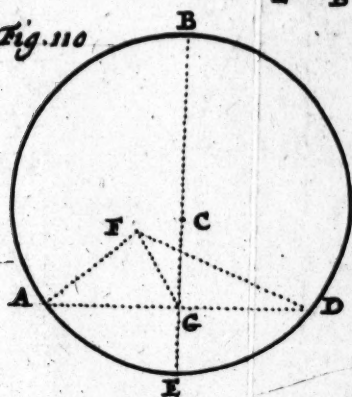


Fig. 111

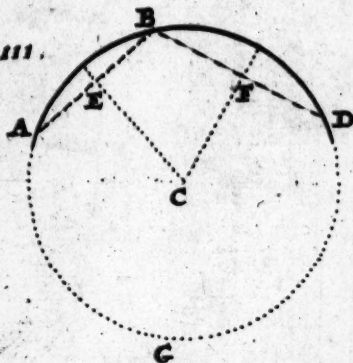


Fig. 112

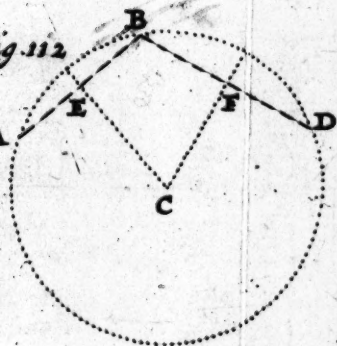


Fig. 113

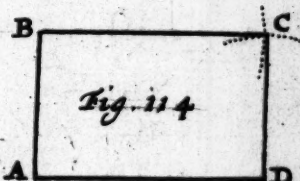


Fig. 114

Fig. 117

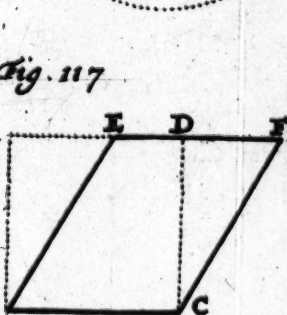


Fig. 115

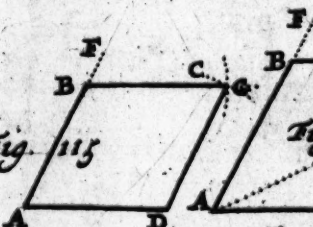


Fig. 116



Fig. 118

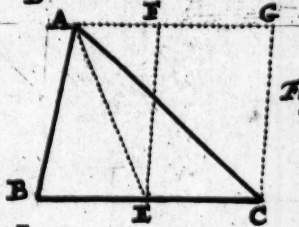


Fig. 119

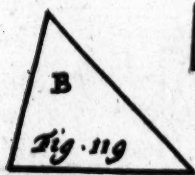


Fig. 120

A

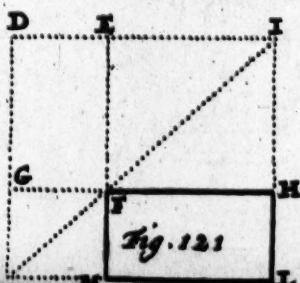


Fig. 121

Fig. 122

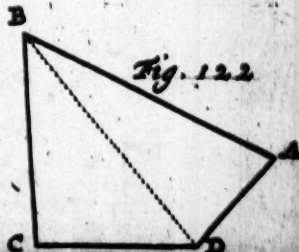








Fig.

1

2

P



Fig. 132

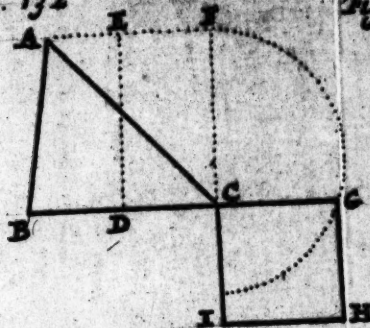


Fig. 133



Fig. 134

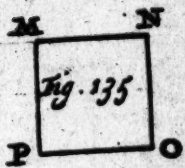
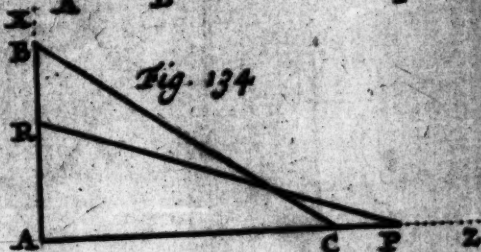


Fig. 135

Fig. 136

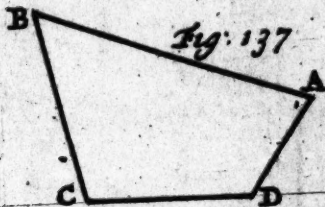
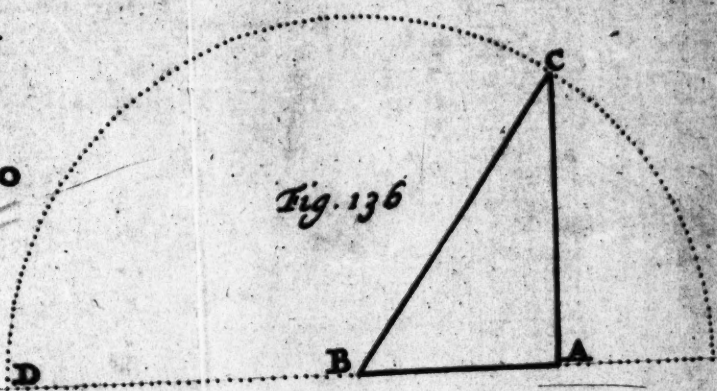


Fig. 137

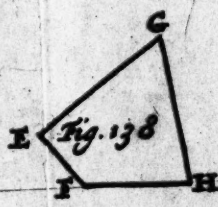


Fig. 138

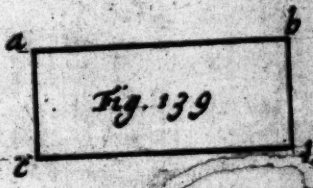


Fig. 139

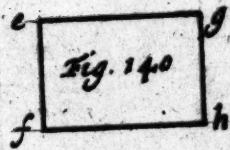


Fig. 140

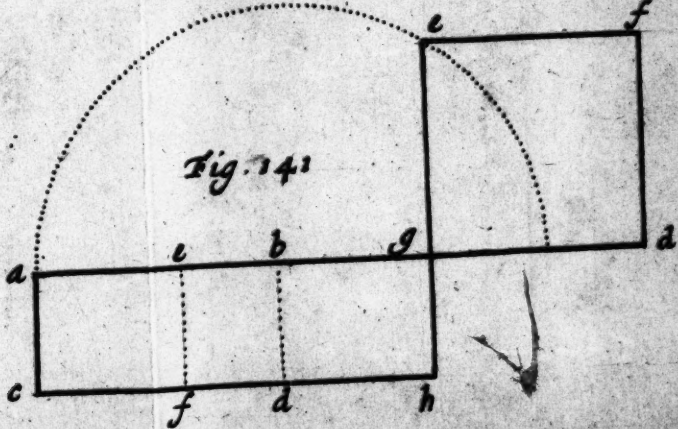
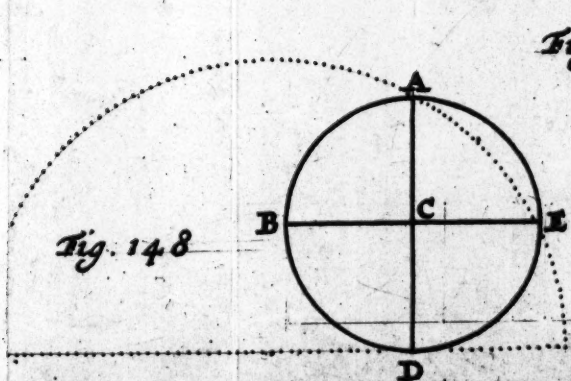
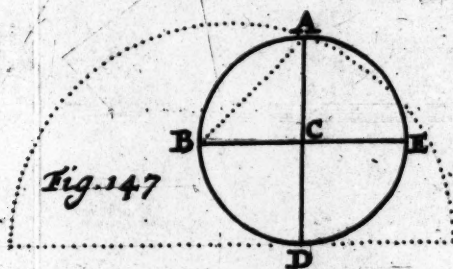
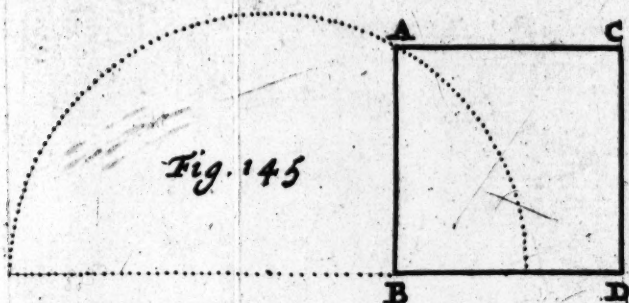
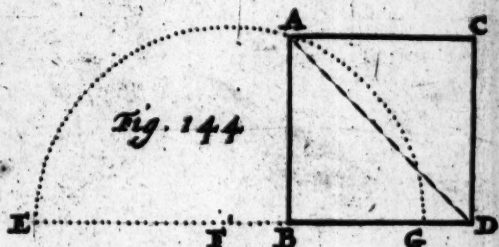
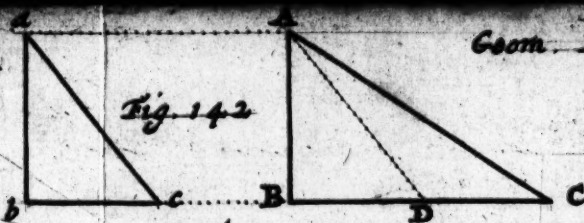


Fig. 141







H

F

E

I

A

C

A

B

I

